C.S.E. (MAIN)
PHYSICS—2008
(PAPER–I)

Time allowed : 3 hours *Max. Marks : 300*

INSTRUCTIONS

Each question is printed both in Hindi and in English.

Answers must be written in the medium specified in the Admission Certificate issued to you, which must be stated clearly on the cover of the answer-book in the space provided for the purpose. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.

Candidates should attempt Question Nos. 1 and 5 which are compulsory, and any three of the remaining questions selecting at least one question from each Section.

Assume suitable data if considered necessary and indicate the same clearly.

All questions carry equal marks.

SECTION 'A'

Q. 1. Answer all the *six* below : 6×10=60

(a) A force field is given by

$$\bar{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}.$$

Is it a conservative field ? If so, what is the scalar potential ?

(b) Show that the Bulk modulus K, Young's modulus Y and Poisson's ratio σ are connected by the relation

$$K = \frac{Y}{3(1-2\sigma)}.$$

(c) The length of a moving rod can be defined as the product of its velocity and the time interval between the instants that both the end points of the rod pass a fixed mark in S system. Show that this definition leads to the space contraction.

(d) Consider a plane electromagnetic wave entering a rarer medium from a denser medium. Show that the Brewster angle is less than the total internal reflection angle.

(e) Find the phase and group velocities for the wave

$$E(z, t) = E_0 \cos(k_1 z - \omega_1 t) + \cos(k_2 z - \omega_2 t)$$

where $|k_1 - k_2| << k_1 k_2$; $|\omega_1 - \omega_2| << \omega_1, \omega_2$.

(f) Explain how inversion of population is achieved in a He-Ne laser.

Q. 2. (a) Derive an expression for the radial and transverse components of the acceleration of a particle moving in a plane. What inferences would you draw regarding the angular momentum if the transverse acceleration is zero? 15

(b) Derive an expression for the moment of inertia of a rigid body about any axis. What is an "ellipsoid of inertia"? Explain clearly what you mean by the terms "principal axes" and "principal moments of inertia"?

Find the moment of inertia of a thin rectangular lamina about an axis passing through the centre of the lamina and perpendicular to its plane. Hence determine the moments of inertia about axes passing through the midpoints of its both sides and perpendicular to its plane. 30

(c) What do you understand by streamline motion and critical velocity of a viscous liquid through a capillary tube. Capillaries of lengths l, $2l$ and $\frac{l}{2}$ are connected in series. Their radii are $r, \frac{r}{2}$ and $\frac{r}{3}$ respectively. If the streamline flow is maintained and the pressure across the first capillary is P_1, deduce the pressures across the second and the third capillaries. 15

Q. 3. (a) A meson of rest mass π comes to rest and disintegrates into a muon of rest mass μ and a neutrino of zero rest mass. Show that the kinetic energy of motion of the muon is

$$T = \frac{(\pi - \mu)^2 c^2}{2\pi}.$$ 20

(b) A convex lens of focal length 20 cm and a concave lens of focal length 10 cm are placed 20 cm apart. In between them an object is placed at a distance x from the convex lens. What is the value of x in cm so that the images formed by both of them coincide ? 20

(c) Let the two waves with parallel electric fields be given by

$$E_1 = 2\cos\left(\vec{k}_1 \cdot \vec{\partial} - \omega t + \frac{\pi}{3}\right) kV/m,$$

$$E_2 = 5\cos\left(\vec{k}_2 \cdot \vec{\partial} - \omega t + \frac{\pi}{3}\right) kV/m.$$

Find the intensity of each beam I_1, I_2 and also the interference term I_{12} at a point where their path difference is zero. Calculate the visibility $V\left(= \frac{I_{max} - I_{min}}{I_{max} + I_{min}}\right)$ for the interference pattern. [$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$, $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$] 20

Q. 4. (a) Calculate the Fraunhoffer diffraction pattern from a grating of N slits with width e, separated by equal opaque spaces d. Find the condition for principal maxima and the corresponding values of intensity.

A parallel beam of Na light is incident normally on a plane grating with 4250 lines per cm. The second order spectral line is observed to be deviated through 30°. Calculate the wavelength of light. 20

(b) In the Lorentz model for dispersion, the uniform dielectric medium is assumed to be a collection of oscillators with number density N, natural frequency ω_0 and decay constant γ. Find the complex susceptibility χ of such a medium. In the frequency dependence of the real part of χ identify the regions of normal and anomalous dispersion. 20

(c) Consider an ensemble of two-level atoms in thermal equilibrium. Show that the ratio of Einstein A and B coefficients is given by

$$\frac{A}{B} = \frac{8\pi h \upsilon^3}{c^3}.$$

Why is it not possible to achieve inversion of population in a two-level medium ? 20

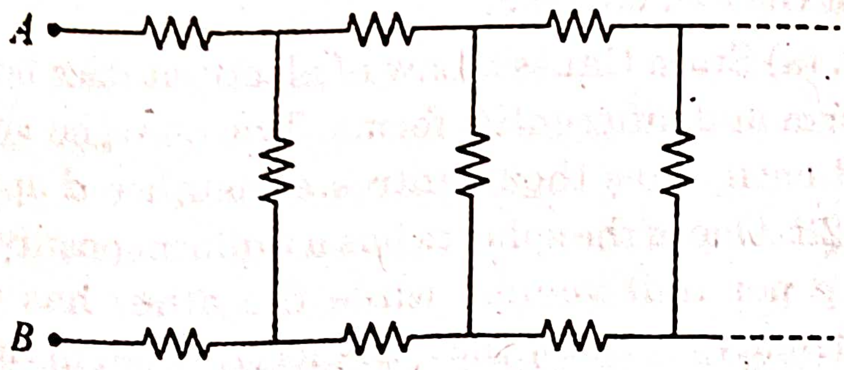
SECTION 'B'

Q. 5. Answer the following (all *six*) : 6×10=60

(a) A point charge $+q$ is held above a grounded conducting plane located at $z = 0$. If the position of the charge is $(0, 0, d)$ obtain an expression for the induced charge density on the plane as a function of coordinates x and y.

(b) State Faraday's Law of induction in terms of emf and magnetic flux. Use Stoke's theorem to express the law in differential form. A uniform, time varying magnetic field $\vec{B} = \vec{B}_0(1 + \alpha t)\hat{k}$ fills a circular region of radius R lying in the x-y plane with the centre at origin. Here $\vec{B}_0$ and α are appropriately dimensioned constants. Find the induced electric field at a distance r from the centre where $r < R$.

(c)



An infinite ladder network of resistances is connected across the points A and B, as shown in the above figure. Each of the resistance is 1Ω. Calculate the effective resistance between points A and B.

(d) Two identical black bodies A and B are respectively at temperatures T and 2T respectively. The heat energy radiated by A is collected for one minute and is used to heat a mass of water. The temperature of water is found to rise by 0.25 K. If the heat radiated by B were to be collected for one minute and used to heat the same mass of water, what would be the rise in the temperature of water ? Assume, specific heat of water to be temperature independent.

(e) One mole of an ideal gas is compressed at constant temperature T from a volume V_1 to a volume V_2. Find the work done and heat absorbed by the gas. The gas now expands adiabatically to a volume 2V. Taking the gas to be diatomic, calculate the final temperature of the gas.

(f) Consider N independent particles, each of which can be in either of two energy states $+\epsilon$ and $-\epsilon$. Derive an expression for the entropy of a microcanonical ensemble for this system, where $f = \Sigma_i \epsilon_i / N\epsilon$ is fixed, where ϵ_i is the energy of the *i*-th particle.

Q. 6. (a) State Gauss's Law of electrostatics both in integral form and differential forms. Two charged spheres of radius R each, have their centres a distance *d* apart such that $d < 2R$. One of the spheres has a uniform positive charge density ρ per unit volume while the other has opposite charge density $-\rho$. Show that the electric field in the region of overlap between two spheres is uniform. 20

(b) Show that the interaction energy of two magnetic dipoles $\vec{m}_1$ and $\vec{m}_2$ separated by a displacement $\vec{r}$ is given by

$$U = \frac{\mu_0}{4\pi} \cdot \frac{1}{r^3}\left[\vec{m}_1 \cdot \vec{m}_2 - 3(\vec{m}_1 \cdot \hat{r})(\vec{m}_2 \cdot \hat{r})\right].$$

If two magnetic dipoles are held at a fixed distance apart, but allowed to rotate freely, what would be the configuration for stable equilibrium ? 20

(c) Find the vector potential due to a line segment from $x = a$ to $x = b$ carrying a current I at a point P which is at a distance d from the line segment. 20

Q. 7. (a) A plane electromagnetic wave is travelling in vacuum along a direction which makes an angle of 45° each with the positive x and z-axes. The electric field at time t is given by the expression

$$\vec{E} = E_0(2 \cos \omega t\hat{j} - \sin \omega t(\hat{i} - \hat{k}))$$

where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the x, y and z directions respectively and $E_0 = 100\ v/m$. Find the magnetic field vector $\vec{B}$ at time t and obtain the average power transmitted per unit area by the wave. 15

(b) Explain how Maxwell modified Ampere's law of magnetostatics by introducing the concept of displacement current. How does it resolve the paradox of a charging capacitor ? 15

(c) A plane electromagnetic wave travelling in z-direction and polarized in x-direction is incident normally from left on to an interface between two media at $z = 0$. The medium to the left ($z < 0$) has a refractive index n_1, while that to the right is of refractive index n_2. The magnetic permeabilities of both the media are equal and $\mu_1 = \mu_2 = \mu_0$. Obtain expressions for reflection and transmission coefficients (R and T) and show that $R + T = 1$. 30

Q. 8. (a) State second law of thermodynamics. Prove that no engine operating between two given temperatures is more efficient than a carnot engine operating between same two temperatures. 20

(b) State the law of equipartition of energy. Show how this law can be used to calculate specific heat of gases and hence find the ratio $\gamma = C_p/C_v$ for diatomic and triatomic gases. 20

(c) Show that at $T = 0$, the Fermi distribution function has a value 1 for energies less than the Fermi energy ε_F and is zero above it. For a system of non-interacting electrons at $T = 0$, show that the ground state energy of the system of N particles is $\frac{3}{5} NE_F$. 20

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The number of marks carried by each question is indicated at the end of the question.

Assume suitable data if considered necessary and indicate the same clearly. Some constants are given at the end of questions.

SECTION 'A'

Q. 1. (a) Show that for the one dimensional wave function

$$\psi(x) = \begin{cases} \dfrac{1}{\sqrt{2a}} & , \ |x| < a \\ 0 & , \ |x| > a, \end{cases}$$

where a is a real constant, the rms uncertainty in momentum is infinite. 10

(b) An electron is in the spin state $\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$.

Determine the normalization constant A. Find the expectation value of the spin operator $\hat{S}_x$ and also the uncertainty in the value of S_x in this state. 10

(c) Write (do not derive) the formula for the energy levels of a particle in a three dimensional cubical box of side L. How many electrons can occupy the level having energ $66h^2/8mL^2$? 10

(d) Explain normal and anomalous Zeeman effect. Obtain expression for Zeeman splitting of an alkali metal spectral line, and illustrate with an example. 10

(e) Draw the potential energy of a diatomic molecule as a function of interatomic distance. Mark the vibrational and rotational energy levels. Explain the selection rule for transition between vibrational states. 10

(f) For transition to the ground state what is the longest wavelength that can be emitted by hydrogen ? 10

Q. 2. (a) The Hamiltonian of a particle moving along the x-axis is given by

$$\hat{H} = -\alpha \frac{d^2}{dx^2} + 16\alpha \hat{x}^2,$$

where α is a real and positive constant having dimensions of energy.

(i) If $\psi(x) = Ae^{-2x^2}$, find the normalization constant A. Check whether ψ is an eigen function of $\hat{H}$. If yes, find the corresponding eigen value.

(ii) Calculate the probability of finding the particle anywhere along the negative x-axis.

(iii) Find the eigen value of $\hat{H}$ corresponding to the eigen function $\phi(x) = x\psi(x)$, where $\psi(x)$ is the same as in part (i).

(iv) Are the wave functions $\psi(x)$ and $\phi(x)$ orthogonal ?

30

(b) Show that the probability of transmission across the step barrier represented by the potential

$$V(x) = \begin{cases} 0 \text{ for } x < 0 \\ V_0 \text{ for } x > 0 \end{cases} \quad \text{is} \quad T = \frac{4k_1k_2}{(k_1 + k_2)^2},$$

where k_1 and k_2 are wave numbers in regions $x < 0$ and $x > 0$, respectively.

30

Q. 3. (a) Write the commutation relations of angular momentum operator $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ and calculate the commutators $[\hat{L}_+, \hat{L}_z]$ and $[\hat{L}_-, \hat{L}_z]$. Show that

$$\hat{L}_+ | l,m = \sqrt{l(l+1) - m(m+1)} \, | l,m+1 >,$$

where $| l, m >$ is the state with definite values for L^2 and L_z.

20

(b) Assume a diatomic molecule consisting of two atoms of masses m_1 and m_2, separated by a distance $\vec{r}$. Write down the Hamiltonian operator for the molecule. Determine the rotational energy levels of the molecule. 20

(c) An electron is moving freely in a one-dimensional infinite potential box with walls at $x = 0$ and $x = a$. If the electron is initially in the ground state of the box and if suddenly the wall at $x = a$ is moved $x = 4a$, calculate the probability of finding the particle in the ground state of the new box. 20

Q. 4. (a) What do you understand by Raman effect ? Explain with help of a diagram, how it can be observed experimentally. 20

(b) On the basis of electronic spectra of molecules explain, how fluorescence and phosphorescence occur. Distinguish between the two. 20

(c) Write full form of acronyms EPR and NMR. Give underlying principle of EPR. 20

SECTION 'B'

Q. 5. (a) Explain why the decay process

$$n \rightarrow p + e^- + \bar{\upsilon}$$

is observed but the decay

$$p \rightarrow n + e^+ + \upsilon$$

is not observed. 10

(b) Which of the following α decay processes is expected to have a large Q value ?

$$^{84}P_0^{211} \rightarrow {}^{82}P_0^{207} + \alpha$$

$$^{84}P_0^{212} \rightarrow {}^{82}P_0^{208} + \alpha$$

Which of these processes will have a longer lifetime ? Explain your answer briefly. 10

Hafnium (Hf) and Tantalum (Ta)

(c) What is nuclear isomerism ? Give two examples and explain how this can be understood from single particle shell model. 10

(d) Show that FCC-crystals are more closely packed than BCC-crystals, 10

(e) How can the NAND-gates be combined to perform the OR-operation ? 10

(f) Explain the Debye model of specific heat of solids. What are its success and failures ? 10

Q. 6. (a) What is β-decay ? Give three examples of β-decay of nuclei. Explain how neutrino hypothesis helped in explaining conservation of energy-momentum and angular momentum. What is Kurie plot ? How can one use Kurie plot to set limits on mass of the neutrino ? 20

(b) Explain lepton number conservation and why it is necessary to distinguish between different types of neutrinos ? 20

(c) Give the hypercharge and isospin of the quarks and antiquarks. What are the quark contents of the mesons: 20

$\pi^+, \pi^0, \pi^-, K^+, K^0, K^-$ and η?

(a) (i) "Transistors are current operated devices, while vacuum tubes are voltage operated."— Explain. 10

(ii) Why are junction transistors called bipolar devices ? 10

(b) A certain colpits oscillator uses a tank circuit with

L = 20 mH; C_1 = 200 pf and C_2 = 300 pf. What is the frequency of oscillation ? 20

(c) Distinguish between 'soft' and 'hard' super-conductors. Explain how penetration depth varies with magnetic field strength and temperature. 20

Q. 8. (a) Draw the circuit diagram of a two-stage RC coupled common emitter transistor amplifier.

Show how the magnitude and phase of voltage gain vary with frequency. Define bandwidth of this amplifier.

An amplifier with open loop voltage gain, A_v = 1000 ± 100 is available. It is necessary to have an amplifier whose voltage gain varies by no more than ± 0.1%. Find the reverse transmission factor β of the feedback network used and the gain with feedback. 25

(b) Explain the switching action of a FET. Two JFETs are connected in parallel with γ_{d1} (drain resistance) = 20 $k\Omega$, γ_{d2} = 30 $k\Omega$ and g_{m1} (transconductance) = 2 m-mho and g_{m2} = 4 m-mho. Find the efffective γ_d and g_m. 20

(c) Show the volt-ampere characteristics of a solar cell under illumination. Explain the characteristics. 15

Constants which may be needed

Velocity of light in vacuum (c) = 3×10^8 m s^{-1}

Mass of electron (m_e) = 9.11×10^{-31} kg

Charge of electron (e) = 1.602×10^{-19} C

Specific charge of electron $\left(\frac{e}{m_e}\right) = 1.76 \times 10^{11}$ C kg^{-1}

1u ≡ 1 a.m.u. = 1.6605×10^{-27} kg = 931.5 MeV

Rest mass energy of electron $(m_e c^2) = 0.5110$ MeV

Permittivity in free space $(\varepsilon_0) = 8.8542 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

Permeability of free space $(\mu_0) = 4\pi \times 10^{-7}\ NA^{-2}$

Gas constant (R) $= 8.314\ J\ mol^{-1}\ K^{-1}$

Boltzmann constant $(k_B) = 1.381 \times 10^{-23} JK^{-1}$

Planck constant $(h) = 6.626 \times 10^{-34}$ Js

$(\hbar) = 1.0546 \times 10^{-34}$ Js

Bohr magneton $(\mu_B) = 9.274 \times 10^{-24} JT^{-1}$

Nuclear magneton $(\mu_N) = 5.051 \times 10^{-27} JT^{-1}$

Fine structure constant $(\alpha) = 1/137.0388$

Mass of proton $(M_p) = 1.0072766$ u $= 1.6726 \times 10^{-27}$ kg

Mass of neutron $(M_n) = 1.0086652$ u $= 1.6749 \times 10^{-27}$ kg

Mass of deuteron $(M_d) = 2.013553$ u

Mass of α-particle $(M_\alpha) = 4.002603$ u

Mass of $^{12}_{6}C$ = 12.000000 u

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