

MEASUREMENT

Static characteristics = accuracy, Resolution, precision (consistency), sensibility

$$\% \text{ accuracy} = 1 - \left| \frac{Y_n - X_n}{Y_n} \right|, \quad |Y_n - X_n| = \text{abs. error}$$

$$|Y_n - X_n| / Y_n = \% \text{ error}$$

$$\text{Precision} = 1 - \left| \frac{X_n - \bar{X}_{avg}}{\bar{X}_{avg}} \right|$$

Errors: Gross (Human), Systematic (Instrumental, Environmental, Observational (1/ax)), Random.

Algebraic sum of the deviations = 0.

$$\text{Deviation avg} = \frac{\sum |D_i|}{n}, \quad \sigma = \sqrt{\frac{\sum d_i^2}{n}}$$

Standards: International > Primary > Secondary > Working

Ω : column of Hg mass 4.4521 gm, length = 106.3 cm at 0°C

Amp = AgNO_3 in water deposits Ag @ rate of 0.0011800 gm/s.

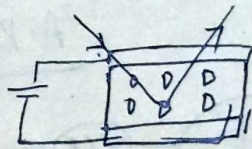
Ω : $\rightarrow$ constant current produce force = $2 \times 10^7 \text{ N/m}$.

DISPLAYS

Active / Passive, Analog / Digital; Symbolic / Console / Large Screen

LCD: low power, light scattering, reflective / transmissive, depend on back light.

Nematic / Cholesteric (NLC)



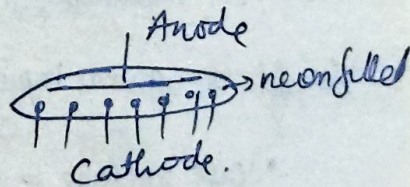
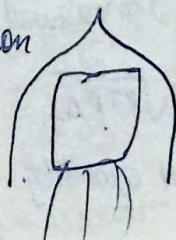
$E = 10^4 \text{ V/cm}$, $\rho > 10^{10} \Omega$, power $\sim \mu\text{W/cm}$, slow

Gas Discharge Plasma: α -numeric; emission of light in cold.

Cathode gas filled tube in breakdown.

Nixies = numerical + Neon

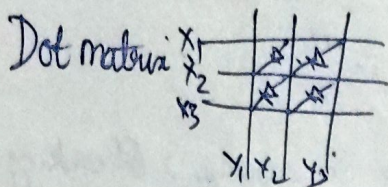
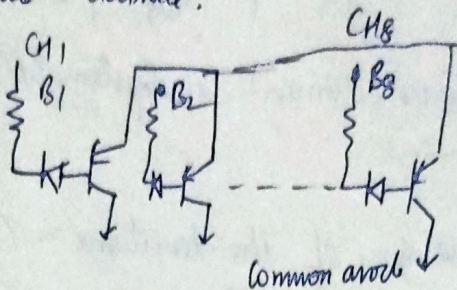
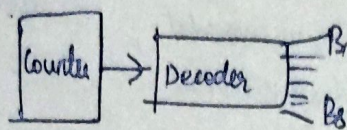
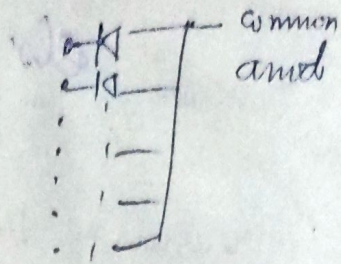
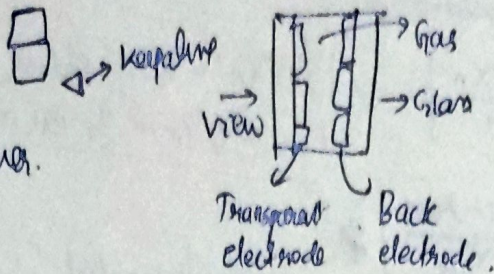
150-220V, 1-5mA



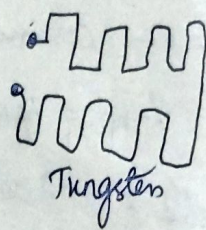
Segmented Gas Discharge ~ Nixie, 7 segment

high voltage, need driving IC. 7 segment. Each segment of each digit has separate Cathode, same anode. "Keep always Cathode".

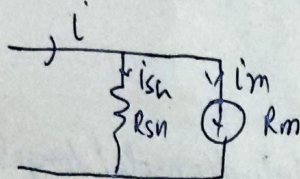
~ μA
~ 180-200V
~ small power.



Bar Graph display (Hydrograph)
Electroluminescent display (phosphor)
Incandescent display



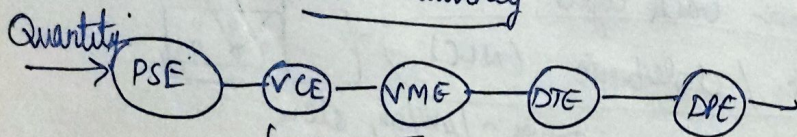
Ammeter



I_m = full scale deflection current
 $I = \dots$ current

$$R_{sh} = \frac{R_m I_m}{I - I_m} = \frac{R_m}{\frac{I}{I_m} - 1} = \frac{R_m}{n - 1}$$

A.K. Sawhney



data conditioning

General instrumentation

Error = measured - true, static error = (- error)

Drift = given input, measured values do not vary with time.

Noise voltage = $2\sqrt{KTR\Delta f}$ $K = 1.38 \times 10^{-23}$

η = measured quantity at full scale / power consumed at full scale

3
5
6
2
3
5
1

Dynamic char

1) Speed of response 2) lag 3) Fidelity 4) Dynamic error

$$N = \begin{cases} > 20, SD = \sigma \\ < 20, SD = s \end{cases}$$

$$\text{else } (s) \rightarrow s = \sqrt{\frac{\sum d^2}{n-1}}, \quad v = \frac{\sum d^2}{n-1}$$

$$\text{Probable error} = 0.6745 \times \text{standard deviation} = 0.6745 \times \sigma$$

$$\text{Probable error of mean} = s_m = \frac{s}{\sqrt{n}}$$

$$\text{Standard deviation of mean} = \frac{\sigma}{\sqrt{n}} = (\sigma_m)$$

$$\text{Standard deviation of standard deviation} = \sigma_\sigma = \frac{\sigma_m}{\sqrt{2}} = \frac{\sigma}{\sqrt{2n}}$$

Q) current $I_1 = 100 \pm 2A$, $I_2 = 200 \pm 5A$. $I = I_1 + I_2$

1) $I_1, I_2 = \text{limiting error} = 300 \pm 7A$

2) standard deviation $\sigma_I = \sqrt{\left(\frac{\partial I}{\partial I_1}\right)^2 \sigma_{I_1}^2 + \left(\frac{\partial I}{\partial I_2}\right)^2 \sigma_{I_2}^2}$

$$\therefore I = 300 \pm 5.38A \quad \sigma_I = \sqrt{2^2 + 5^2} = 5.38$$

Q) $V = 100V$, probable error $= \pm 12V$, $A = 10A$, $I = \pm 2A$ } R probable error.

$$r_{RV} = \frac{\partial r}{\partial V} r_V = I^{-1} r_V = \frac{r_V}{I} = \frac{\pm 12}{10} = \pm 1.2\Omega$$

$$r_{RI} = \frac{\partial r}{\partial I} r_I = \frac{V}{I^2} r_I = \frac{100}{10^2} \times \pm 2 = \pm 2\Omega$$

$$\therefore r = \sqrt{1.2^2 + 2^2} = \sqrt{1.2^2 + 2^2}$$

Q) $I = 10$, $\theta = \pm 0.1^\circ$ (s.d) of 15° , find % s.d. of I

$$I = 10 (\cos \theta)^{-1} \quad \frac{\partial I}{\partial \theta} = K \left\{ \frac{1}{\cos^2 \theta} + \theta \sin \theta (\cos \theta)^{-2} \right\}$$

$$s.d. = \frac{\partial I}{\partial \theta} \cdot \theta = \pm 1.11K \times 0.1 \times \frac{\pi}{180} = A$$

$$\therefore \% = \frac{A}{K \times \pi / 180 \times 15 \times \cos 15} = \pm 0.71\%$$

$$\text{Uncertainty} = \sqrt{\left(\frac{\partial P}{\partial V}\right)^2 \omega_V^2 + \left(\frac{\partial P}{\partial I}\right)^2 \omega_I^2 + \dots}$$

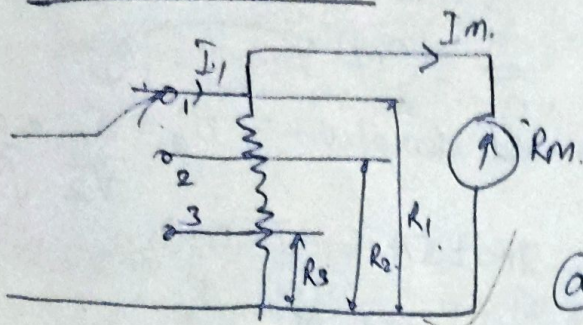
D-arcenal Galvanometer

$$T_{\text{defl}} = (NBA)i = T_{\text{cont}} = k\theta \Rightarrow \theta \propto i$$

(PMMC)

Assuming
series
amp

UNIVERSAL Shunt



extended range.

$$I_1, I_2, I_3. \quad m_1 = \frac{I_1}{I_m} = 1 + \frac{R_m}{R_1}$$

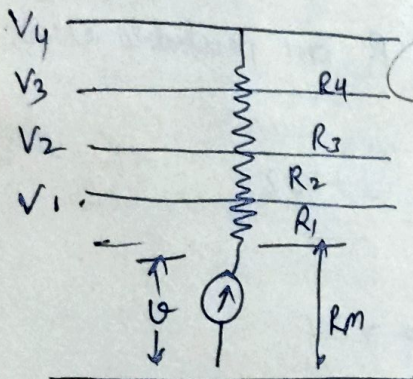
$$\Rightarrow R_1 = \frac{R_m}{m_1 - 1}$$

$$@ 2: I_m(R_1 + R_2 + R_m) = (I_2 - I_m)R_2$$

$$@ 3: I_m(R_1 + R_3 + R_m) = (I_3 - I_m)R_3$$

voltage multipliers

$$m = \frac{V_{\text{rang.}}}{V_{\text{fd}}} \quad R_s = (m-1)R_m$$



$$R_m + R_1 = \frac{V_1}{I_m} \Rightarrow R_1 = \frac{V_1}{I_m} - R_m = \frac{V_1}{I_m} - R_m = (m_1 - 1)R_m$$

$$R_2 + R_1 + R_m = \frac{V_2}{I_m} = \frac{V_2}{I_m} \Rightarrow R_2 = (m_2 - m_1)R_m$$

$$R_3 = (m_3 - m_2)R_m \quad R_4 = (m_4 - m_3)R_m$$

Moving Iron

$$\text{Electrical energy supplied} = eIdt, \quad e = \frac{d}{dt}(LI) = L \frac{di}{dt} + i \frac{dL}{dt}$$

$$\therefore Lidi + i^2 dL$$

$$\text{Change in stored energy} = \frac{1}{2}((I+di)(L+dL) - \frac{1}{2}LI^2)$$

$$= \frac{1}{2}((I^2 + 2IdI)(L+dL) - LI^2)$$

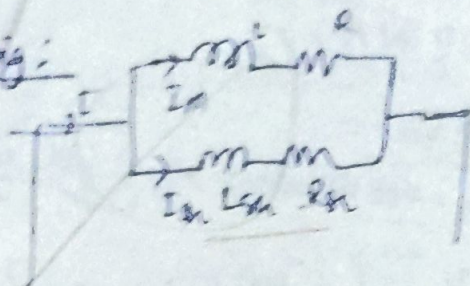
$$\Delta = \frac{1}{2} (I^2 dL + 2LI dI + I^2 dL) = \frac{I^2 dL}{2} + LI dI$$

$$\therefore \frac{I^2 dL}{2} + LI dI = \frac{LI dI}{2} + I^2 dL + T d\theta \Rightarrow T d\theta = \frac{I^2 dL}{2}$$

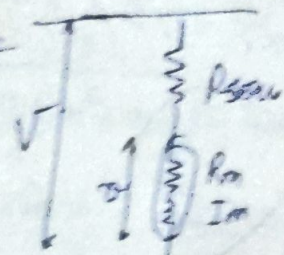
$$\therefore T = T_d \Rightarrow \theta = \frac{1}{2} \cdot \frac{I^2}{k} \cdot \frac{dL}{d\theta}$$

Shunt:

$$\frac{L_{sh}}{I_{sh}} = \frac{L}{I}$$

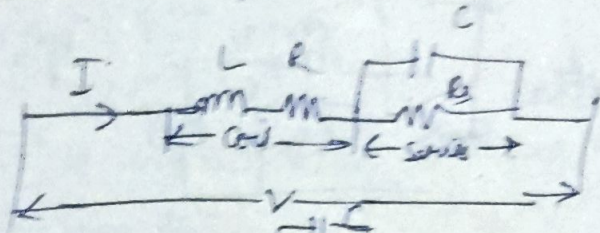


Shunt
Multiplier



Freq. Compensation

make L cancel C

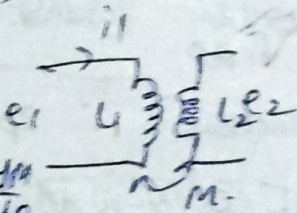


$$L \ll R \ll R_s \quad \text{Objective: } \frac{L}{R} = \frac{R_s}{R}$$

$$\therefore |j\omega L + R_s \parallel 1/j\omega C| = R_s \quad \therefore L = 2.41 C R_s^2 \quad \text{--- (1)}$$

Electrodynamometer

$$I_1 I_2 dM = T d\theta \Rightarrow T = \frac{dM}{d\theta}$$



$$\Rightarrow \theta = \frac{I_1 I_2 dM}{k d\theta}$$

$$\text{if } i_1 = i_m \sin \omega t, i_2 = i_m \sin(\omega t - \phi) \quad T_d = \frac{I_{m1} I_{m2} \cos \phi}{2} \frac{dM}{d\theta} = \frac{(I_{rms})^2 \cos \phi}{k} \frac{dM}{d\theta}$$

Ammeters : $I_1 = I_2 = I, \phi = 0$

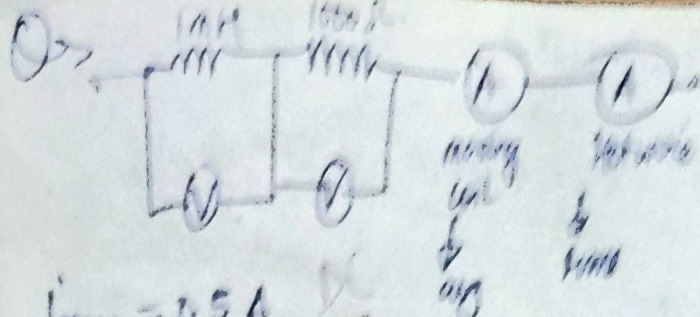
$$\therefore \theta = \frac{I^2 dM}{k d\theta}$$

Voltmeter $I_1 = I_2 = \frac{V}{Z}, \phi = 0 \therefore \theta = \frac{V^2}{k Z^2} \frac{dM}{d\theta}$ (freq. indep)

Thermocouple : $E_{mf} = a(\Delta T) + k(\Delta T)^2 \quad \theta = k I^2$

freq. independent : make $\tau = \frac{L}{R}$ same.

$$k \theta = \frac{V^2}{2} \frac{dC}{d\theta}$$



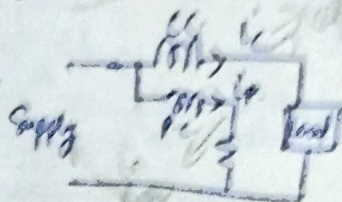
$$I_{me} = 0.5A$$

$$I_{Hot} = \sqrt{0.5^2 + \frac{(0.3)^2}{2} + \frac{(0.2)^2}{2}} = 0.61A$$

$$V_{load, 1000\Omega} = 0.61A \times 1000 = 615V$$

$$V_{Limn} = L \frac{di}{dt} = \frac{200 \times 10^{-3} \times 1000}{25} = 8V$$

Wattmeter



$$T = I_p I_c \cos \phi$$

I_p is the current through the pressure coil
 I_c is the current through the current coil

Now, PC has very high R $\therefore$ neglect its L . $\therefore I_p = \frac{V}{R_p}$

$$I_{cc} = \sqrt{2} I \sin \phi \quad \therefore T = 2 I_p I \sin \phi \cos \phi$$

$$\therefore \tan \phi = \frac{I_p I \cos \phi}{T} \quad (2.11) \quad \tan \phi = \frac{V}{R_p I}$$

$$\textcircled{1} \text{ Pressure coil inductance} \quad \therefore Z_p = \sqrt{R_p^2 + X_p^2} = \frac{L_p R_p}{\tau_p}$$

$$\therefore T = \frac{V}{Z_p} I \cos (\phi - \beta) \quad \beta = \tan^{-1} \frac{X_p}{R_p} = \tan^{-1} \frac{\omega L_p}{R_p}$$

True Cosine factor = $\frac{\cos \phi \cos \beta}{\cos (\phi - \beta)}$ $\log (+\beta \text{ for lead})$

2 connections of wattmeter

- PC is on supply side $\therefore I_{cc} = I_c$
- CC is on supply side $\therefore I_{cc} = V/R_p$

$$\% \text{ error} = \frac{\sin \phi}{\sin \beta + \cos \phi} \quad \phi = \text{power factor}$$

- Errors: 1) inductance 2) H 3) eddy
4) capacitance 5) connection 6) battery
7) vibration of moving 8) temp

Wattmeter: 10A, 25V. $R_c = 0.06 \Omega$, $X_c = 0.02 \Omega$ Kf

(10A, 25V, 0.174 lag)

a) error if C.C. is toward load.

$$P_{\text{act}} = VI \cos \phi = 43.5 \text{ W}$$

$$I R_c = 0.6 \text{ V}, I X_c = 0.2 \text{ V} \quad \cos \phi = 0.174 = 80^\circ$$

$$\therefore V_P = 25 + 0.6 \cos 80^\circ + 0.2 \sin 80^\circ - j(0.6 \sin 80^\circ + 0.2 \cos 80^\circ)$$

$$= 25.3 \angle -1.4^\circ$$

$$\therefore P = 25.3 \times 10 \times \cos(80 - 1.4) = 50 \text{ W} \quad \therefore \text{error} = \frac{50 - 43.5}{43.5}$$

b) error if P.C. toward load $\frac{25^2}{250} = 0.1 \text{ W}$

Reading of wattmeter $\propto \cos \phi \cos(\phi + \beta)$ (loading load)
(do some examples on page 365)

Measurement of Resistance

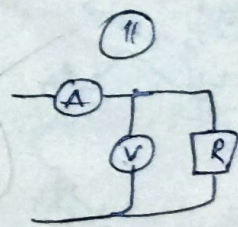
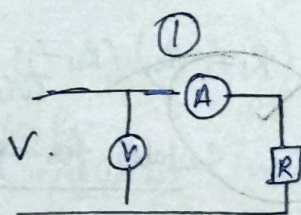
low: 1Ω $1 \Omega - 100000$ Medium > 100000 : High

Medium

1) Ammeter voltmeter $R = \frac{V}{I}$

relative error equal

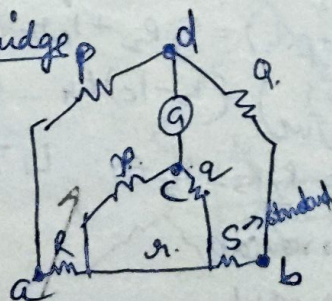
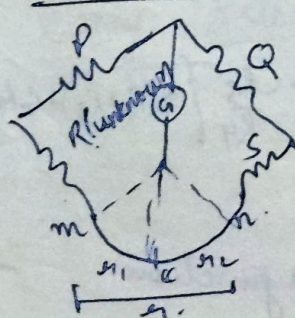
$R = \sqrt{R_a R_v}$ if $R > \sqrt{R_a R_v}$ use ①
(breakdown point) $< \sqrt{R_a R_v}$ use ②



True value = $R_{m2} \left(\frac{1}{1 - R_{m2}/R_v} \right)$, $R_{m2} = \frac{V}{I}$

P, Q = outer arms
p, q = inner arms
S = standard

Kelvin Double Bridge



1st arm = P, Q
2nd arm = p, q

$$R = \frac{P}{Q} S + \frac{q r}{P + q + r} \left[\frac{P - p}{Q} - \frac{r}{q} \right] \text{ in terms of } \epsilon_{ab}$$

@ $\epsilon_{ad} = \epsilon_{ac}$

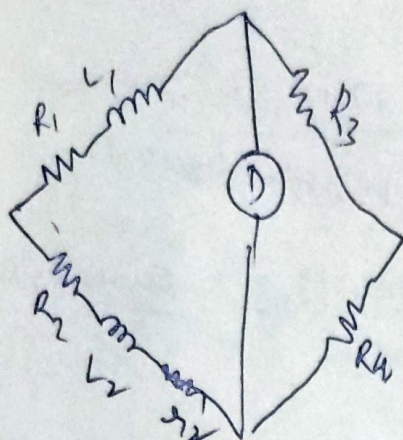
$(R + r_1) Q = P(S + r_2)$
but $r_1 = P/Q \therefore RQ = PS$

$\frac{P}{P+Q} \epsilon_{ab} = I(R + \frac{q r}{P+q+r})$

1 2 3 4

AC BRIDGES

① Maxwell Inductance (Self)



$L_1 = \text{unknown}$

$$L_1 = \frac{R_3 L_2}{R_4} \quad \text{freq independent}$$

$$R_1 = \frac{R_3 (R_2 + R_4)}{R_4}$$

R

Maxwell Induct Cap

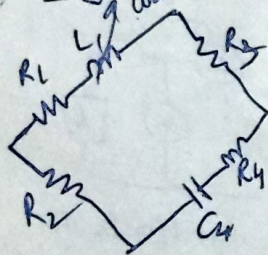
$$(R_1 + j\omega L_1) \left(\frac{R_4}{1 + j\omega C_4 R_4} \right) = R_2 R_3$$

requires standard cap $Q = \frac{\omega L_1}{R_1} = \omega R_4 C_4$

Adv - no freq, independent, simple

Disadv: variac, low Q / medium

② Hay (opposite series)

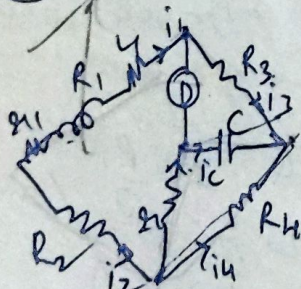


$$(R_1 + j\omega L_1) (R_4 - \frac{j}{\omega C_4}) = R_2 R_3 \quad Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega R_4 C_4}$$

Suitable for High Q: independent of freq, simple

Disadv: Not for low Q $\omega R_4 C_4$ reqd

③ Anderson



$$i_1 = i_3, i_2 = i_c + i_4 \quad \text{also } i_1 R_3 = i_c \cdot \frac{1}{j\omega C} \therefore i_c = i_1 j\omega C R_3$$

$$I_1 (R_1 + R_2 + j\omega L_1) = I_2 R_2 + i_c R_1$$

$$I_c \left(\frac{R_1 + 1}{j\omega C} \right) = (i_2 - i_c) R_4$$

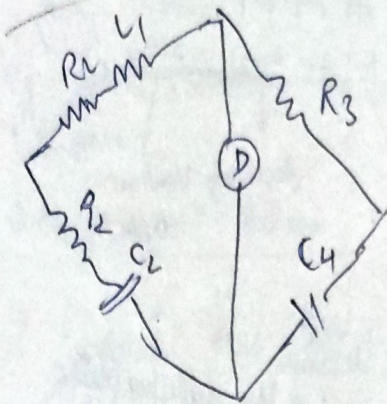
$$\therefore R_1 = \frac{R_2 R_3 - R_1}{R_4} \quad L_1 = \frac{C R_3}{R_4} [R_1 (R_2 + R_3) + R_2 R_4]$$

Adv: i) fixed cap, no variac

ii) Q_1, Q_2 independent

Disa: complex, shielding, extra function

④ Owen



$$(R_1 + j\omega L_1) \frac{1}{j\omega C_4} = \left(R_2 + \frac{1}{j\omega C_2} \right) R_3$$

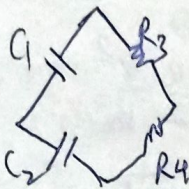
$$\mu = \frac{L_1}{N^2 A}$$

Adv: Independent
Simple
no freq

Disadv: variable cap

Capacitance

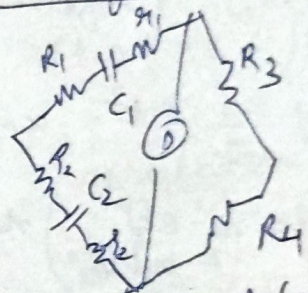
① De Sauty



$$\frac{1}{j\omega C_1} R_4 = R_3 \cdot \frac{1}{j\omega C_2}$$

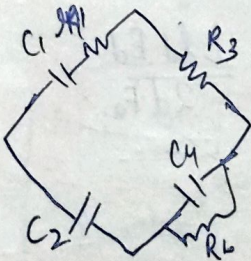
$$\text{Dissipation} = \tan \delta = \omega C_1 R_4$$

modified



$$\left(R_1 + \frac{1}{j\omega C_1} \right) R_4 = R_3 \left(R_2 + \frac{1}{j\omega C_2} \right)$$

② Schering



$$\left(\frac{1}{j\omega C_1} + R_1 \right) \left(\frac{R_4}{1 + j\omega C_4 R_4} \right) = \frac{R_3}{j\omega C_2} \Rightarrow \frac{R_1}{C_2} = \frac{R_3}{C_4}$$

$$C_1 = \frac{R_4 C_2}{R_3}$$

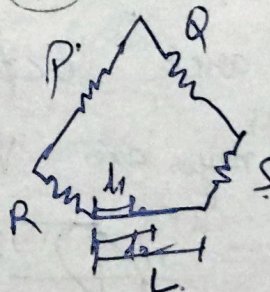
$$\text{dissipation } D_1 = \omega R_4 C_4$$

Storage factor

$$Q = \frac{\omega L}{R}$$

$$\text{Dissipation } D_1 = \tan \delta_1 = \omega R_1 C_1$$

Carey Foster

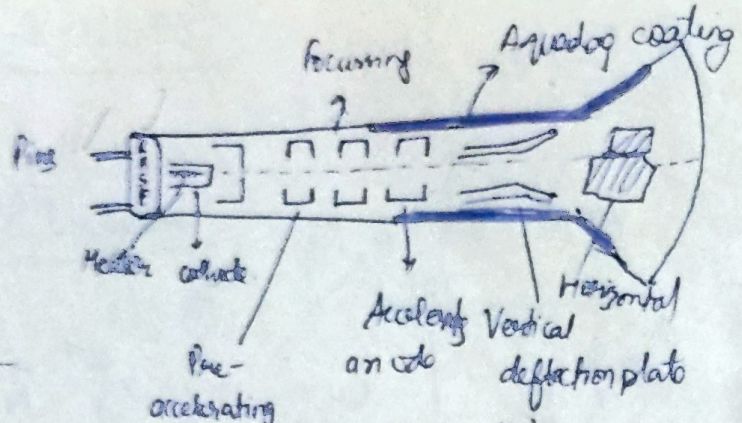
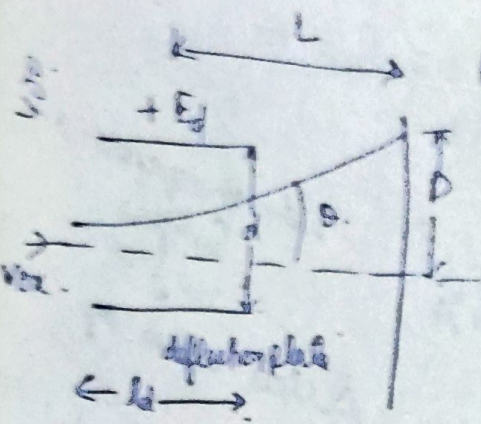


$l_1 = l_2 =$ 1st & 2nd balances

$$\text{1st: } \frac{P}{R + l_1} = \frac{Q}{S + (L - l_1)}$$

$$\text{2nd: } \frac{P}{R + l_2} = \frac{Q}{S + (L - l_2)}$$

CRO



$ld = \text{length}$
 $d = \text{distance}$

$E_a = \text{voltage of pre-accelerating anode}$. $E_d = \text{potential b/w deflection plates}$
 $PE = e \cdot E_a$, $KE = \frac{1}{2} m v_{0x}^2 \Rightarrow v_{0x} = \sqrt{\frac{2eE_a}{m}}$ (entering the deflection)

$$V = E_d/d, F = eE = \frac{eE_d}{d} \therefore a = \frac{eFd}{m}$$

$$y = \frac{1}{2} \cdot a \cdot t^2, x = v_{0x} t \therefore t = \frac{x}{v_{0x}} \therefore y = \frac{a}{2} \cdot \frac{x^2}{v_{0x}^2}$$

$$\Rightarrow y = \left(\frac{1}{2} \frac{eFd}{m \cdot v_{0x}^2} \right) x^2 \text{ parabola} \quad \frac{dy}{dx} = \frac{eFd}{m \cdot v_{0x}^2} \cdot x = ld$$

$$\tan \theta = \frac{D}{L} \Rightarrow D = \frac{LeFd \cdot ld}{m \cdot v_{0x}^2} \quad \tan \theta = \frac{eFd \cdot ld}{m \cdot v_{0x}^2}$$

$$\text{now } v_{0x}^2 = \frac{2eE_a}{m} \therefore D = \frac{LeFd \cdot ld \cdot m}{m \cdot 2eE_a} = \frac{L \cdot ld \cdot E_d}{2d \cdot E_a}$$

$E_d = \text{deflection voltage}$ $\therefore E_a = \text{fixed}$
independent of e/m ratio

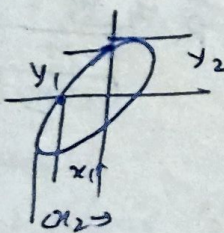
$$\text{also } L \cdot ld \propto \sqrt{\frac{1}{\lambda m v_0}}$$

$$S = \frac{D}{E_d} = \frac{L \cdot ld}{2d \cdot E_a} = \text{sensitivity} \quad \text{Deflection factor } G \geq 1 = \frac{2d \cdot E_a}{L \cdot ld}$$

$\phi = 0$: line $\phi = 90$: circle $0 < \phi < 90$: ellipse

$V_y > V_x$: vertical major axis $V_x > V_y$: horizontal major axis

$$\sin \phi = \frac{y_1}{y_2} = \frac{x_1}{x_2}$$



aquadag = collect secondary emission electrons

$$\frac{f_y}{f_x} = \frac{\text{no. of horizontal tangencies}}{\text{no. of vertical tangencies}}$$

if free ends $\frac{f_y}{f_x} = \text{no. of intersections} \left\{ \frac{H}{V} \right\}$
Dynamic Char treat open end as half tangents

A balloon has 1st order therm of $z = 15^\circ$, 6 m/s , $0.005^\circ/\text{m}$. At 3000 m , balloon says 15° . At what altitude 15° occurred?

$$\frac{dT}{dt} = 0.005^\circ/\text{m} \times 6 \frac{\text{m}}{\text{s}} = 0.03^\circ/\text{s} \text{ (rate)}$$

$$e_{ss} = A\tau \quad (y(t) = A(t - \tau + \tau e^{-t/\tau}))$$

$$= 0.03 \times 15 = 0.45^\circ$$

$$\therefore \text{error} = \frac{0.45 \times 6}{0.03} = 90 \text{ m} \quad \therefore \text{actual} = 3000 - 90 = 2910 \text{ m}$$

Ramp reaction of 2nd order

$$C(s) = \frac{1}{s^2} \left(\frac{\omega^2}{s^2 + \omega^2 + 2\xi\omega s} \right) = 1 - \frac{2\xi}{\omega_n} + \frac{e^{-\xi\omega_n t} \sin[\omega_d t + \cos^{-1}\xi]}{\omega_d}$$

$$\omega_n = \omega_n \sqrt{1 - 2\xi^2} \quad M_s = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\text{BW} = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{(1 - 2\xi^2)^2 + 1}}$$

$$e_{ss} = 2\xi/\omega_n \quad \tau = \frac{1}{\omega_n} \quad T = \frac{2\xi}{\omega_n} \text{ (lag)}$$

$$M = \frac{1}{\sqrt{(1 - u^2)^2 + (2\xi u)^2}} = 0.9/1.1$$

$$u = \frac{\omega}{\omega_n} \quad \phi = -\tan^{-1} \left(\frac{2\xi u}{1 - u^2} \right)$$

Lag of 1st order system

$$T_1 = \frac{1}{\omega} \tan^{-1}(\omega\tau)$$

Phase shift = $-\tan^{-1}(\omega\tau)$

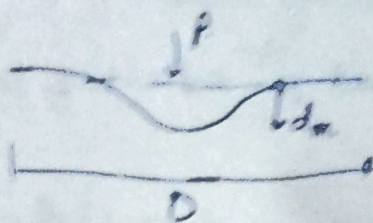
Ratio of ω/ω_n to $i/p = M = \frac{1}{\sqrt{1 + \omega^2\tau^2}} \leftarrow 0.95$

(angle of lag = $\tan^{-1}\omega\tau_1 + \tan^{-1}\omega\tau_2 + \dots$)

Transducers

Shear strain is measure of torque $T = \frac{\pi d^3}{16} \tau$

Deflection



$$\delta = \frac{(254) E t^3 d_m}{3(1-\nu^2) D^3}$$

t = thickness of material
 ν = Poisson's ratio
 E = Young's modulus
 D = dia

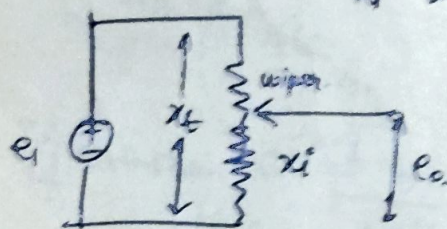
max stress $S_m = \frac{3P^2}{16t^2} \quad \rho = \text{density}$

Sensor: convert to other Transducer electrical

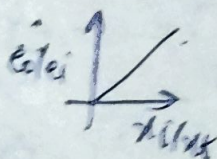
Active: self generating (Pierzo) Passive: load cell

Potentiometer

R_f = total, x_f = total

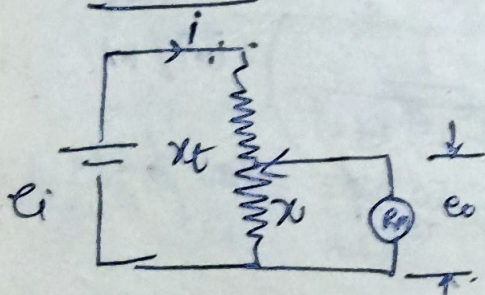


$$e_o = \left(\frac{R_o/x_f}{R_f} \right) e_i = \frac{x_i}{x_f} e_i$$



$$S = \frac{e_o}{x_i} = \frac{e_i}{x_f}$$

Now loaded



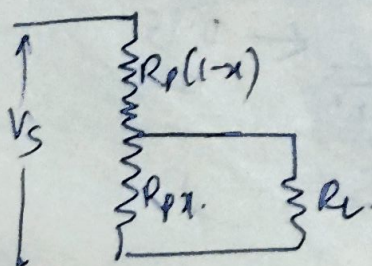
(R_o/R_L)

$$R_{eq} = \frac{k R_p R_L}{k R_p + R_L}, \quad k R_p = \left(\frac{x_i}{x_f} \right) R_p$$

$$\therefore R_{tot} = R_p(1-k) + \frac{k R_p R_L}{k R_p + R_L}$$

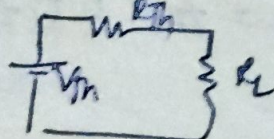
$$i = \frac{e_i}{R_{total}}$$

$$\therefore e_o = e_i - i R$$



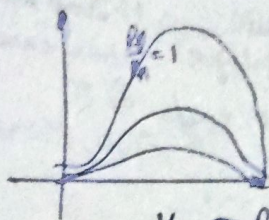
$$e_{th} = V_s x \left(\frac{e_{th}}{V_s} = \frac{R_p x}{R_p} \right)$$

$$R_{th} = R_p(1-x) \parallel R_p(x) = R_p x(1-x)$$

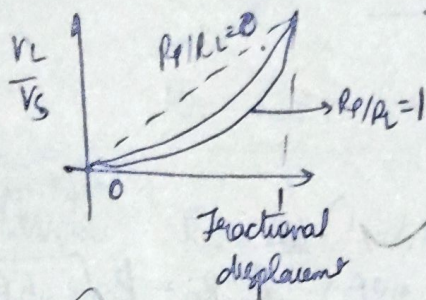


$$V_L = \frac{E_m R_L}{R_L + R_m} = V_s \times \frac{R_L}{R_L + R_p \times (1-x)}$$

$$\text{Non linear error} = N(x) = E_m - V_L = V_s \times \left\{ 1 - \frac{R_L}{R_L + R_p(1-x)} \right\}$$



$$K = \frac{R_p}{R_L}$$



$$\therefore N(x) \approx V_s \cdot \frac{R_p}{R_L} (x^2 - x^3) \quad \text{if } \frac{R_p}{R_L} \ll 1$$

$$\max N(x) = \left(\frac{2}{3} \right) (2x) = \frac{4}{27} V_s \frac{R_p}{R_L}$$

$$\therefore \hat{N} = \frac{400}{27} \frac{R_p}{R_L} \% \quad (\text{as } \% \text{ of s.d.})$$

$$\approx 15 \% \frac{R_p}{R_L}$$

$$\text{Power rating: } \epsilon_{\max} = \sqrt{P \cdot R_p}$$

for good linearity, $R_p \rightarrow 0$
 for good sensitivity, $\epsilon_0 \uparrow \rightarrow \epsilon_i \uparrow$ but $\therefore$ sensitivity $\uparrow$ needs $\epsilon_i \uparrow, R_p \uparrow$
 $\rightarrow$ conflicting

Wire wound, Cermet, Carbon film, Thin metal film

STRAIN GAUGE

$$R = \frac{\rho l}{A} \quad \Delta R = \left(\frac{\partial R}{\partial l} \right) \Delta l + \left(\frac{\partial R}{\partial A} \right) \Delta A + \left(\frac{\partial R}{\partial \rho} \right) \Delta \rho$$

$$\therefore \Delta R = \frac{\rho}{A} \Delta l - \frac{\rho l}{A^2} \Delta A + \frac{\rho}{A} \Delta \rho$$

$$\left| \frac{\Delta l}{l} = e_{\text{longitudinal}} \right|$$

$$\therefore \left| \frac{\Delta R}{R} = \frac{\Delta l}{l} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho} \right| \quad \left(\because \text{by } R = \frac{\rho l}{A} \right)$$

$$A = wt$$

$$\therefore \frac{\Delta A}{A} = \frac{\Delta w}{w} + \frac{\Delta t}{t} = 2e_{\text{trans}}$$

$$\therefore \frac{\Delta R}{R} = e_l - 2e_t + \frac{\Delta \rho}{\rho} \quad (e_l = \Delta l/l)$$

$$\text{Now, } \nu = -\frac{e_t}{e_l}$$

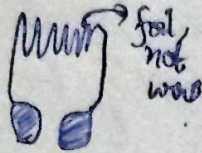
Poisson

$$\frac{\Delta R}{R} = e_l(1+2\nu) + \frac{\Delta \rho}{\rho}$$

$$\therefore Gf = \frac{\Delta R/R}{e_l} = \underbrace{(1+2\nu)}_{\text{length area}} + \frac{\Delta \rho}{\rho \cdot e_l}$$

piezoresistive

i) wire $\begin{cases} \text{bonded} \rightarrow \text{Linear, Potentiometer, Helical, Torque} \\ \text{unbonded} \end{cases}$

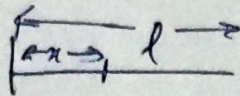
ii) Foil (metal)  foil not wire

iii) semiconductors

Adv $\rightarrow$ +130, hysteretic, small, no fatigue

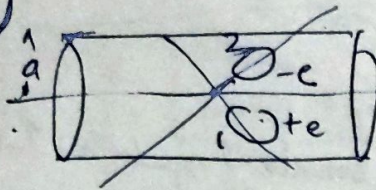
Disadv
1) Temp sensitive
2) nonlinearity
3) π

$\Rightarrow R = 120 \Omega$, steel cantilever 0.15m from free end (wait)

① $e = \frac{6(l-x) \cdot F}{wt^2 E}$ 

WFS

② $e_t = -\frac{F}{AE}$ 
 $e_r = +\frac{\nu F}{AE}$ $R_1 R_3 = R_0 \left(1 + \frac{\nu F}{AE}\right)$ $R_2 R_4 = R_0 \left(1 - \frac{\nu F}{AE}\right)$

③  $e = \frac{I}{\pi S a^3}$ $R_1 R_3 = R_0 (1 + e)$ $R_2 R_4 = R_0 (1 - e)$

$\Rightarrow R = 120 \Omega$, steel cantilever 0.15m from free end. deflection of 12.7m of free end. $\Delta R = 0.152 \Omega$. $L = 0.25m$,

width = 20mm depth = 3mm $\gamma = 200 \text{ GN/m}^2$ Calculate γ

$I = \frac{1}{12} bd^3 = \text{mom of inertia} = \frac{1}{12} \times 0.02 \times (0.003)^3 = 45 \times 10^{-12}$

$x = \text{deflection} = \frac{Fl^3}{3EI}$ $\Rightarrow F = \frac{3EIx}{l^3} = \frac{3 \times 200 \times 10^9 \times 45 \times 10^{-12}}{(0.25)^3}$

Bending moment $M = Fx = 22 \times 0.15 = 3.3 \text{ Nm}$

stress $= \frac{M}{I} \cdot \frac{t}{2} = \frac{3.3}{45 \times 10^{-12}} \times \frac{0.003}{2} = 110 \text{ MN/m}^2$

$E = \frac{\Delta L}{L} = \frac{S}{E} = \frac{110 \times 10^6}{200 \times 10^9} = 0.55 \times 10^{-6}$

$\Delta R = \frac{\Delta R/R}{\Delta L/L} = \frac{0.152/120}{0.55 \times 10^{-6}} = 23$

RTD $R_T = R_0 (1 + \alpha_1 \Delta\theta + \alpha_2 (\Delta\theta)^2)$

Temp	90	95	100	105	110	115	120	125	130
R_T	562.66	568.23	573.40	578.77	584.13	589.44	594.84	600.18	605.52

1) Linear appⁿ b/w 100 & 130, mean 115^o

$$R_T = R_0 (1 + \alpha \Delta\theta) \quad \alpha = \frac{1}{589.48} \frac{605.52 - 573.40}{30^o} = 0.00182$$

$$R_0 = 589.48 (1 + \alpha (0 - 115))$$

2) Quadr $R_0 = R_{00} (1 + \alpha_1 \Delta\theta + \alpha_2 (\Delta\theta)^2)$

$$573.4 = 589.48 (1 + \alpha_1 (100 - 115) + \alpha_2 (100 - 115)^2)$$

$$605.52 = 589.48 (1 + \alpha_1 (130 - 115) + \alpha_2 (130 - 115)^2)$$

↓ DC
→ disc, rod, bead, washer

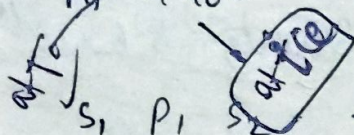
Thermistor
NTC

$$R_{T1} = R_{T2} \exp \left[\beta \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \right]$$

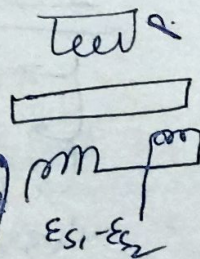
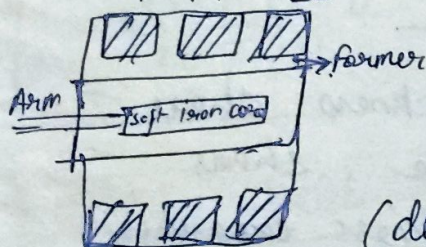
Kelvin

metallic oxides
Mn, Ni, Co, Cu,
of P.

$$\sim R_T = R_0 e^{\beta/T}$$

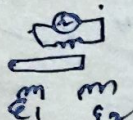


LVDT

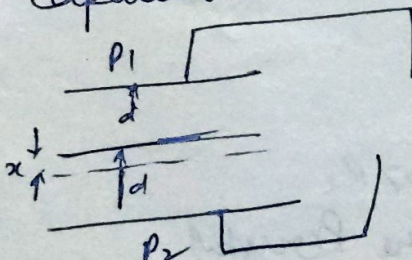


(differe)

$$E_0 = E_1 - E_2$$



Capacitive

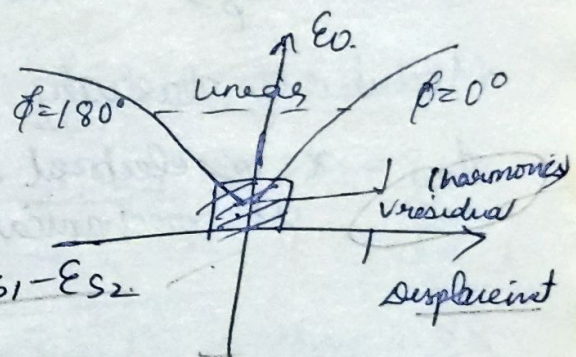


$$C_1 = \frac{A\epsilon}{d}, C_2 = \frac{A\epsilon}{d}$$

$$\text{Now } C_1 = \frac{A\epsilon}{d-x}, C_2 = \frac{A\epsilon}{d+x}$$

$$\therefore \Delta E = E_1 - E_2 = \frac{x}{d} \cdot E$$

$$\text{Sensitivity} = \frac{\Delta E}{x} = E/d$$



Adv

linear, no resd, high o/p, sens, Rugged, ↓ frictn, ↓ hydro, ↓ w

Disad

stray B, vibration, A.C. only

$$E_1 = \frac{\epsilon C_2}{C_1 + C_2}, E_2 = \frac{\epsilon C_1}{C_1 + C_2}$$

$$E_1 = \frac{d-x}{2d} E, E_2 = \frac{d+x}{2d} E$$

Cylindrical Cap: $\frac{2\pi \epsilon l}{\ln(D_2/D_1)}$ $l = \text{length of electrode}$

$M = \frac{zw}{\sqrt{1+zw^2}} = \frac{1}{\sqrt{1+\frac{1}{zw^2}}}$ $\phi = \frac{\pi}{2} - \tan^{-1} wt$

Piezo Rochelle Salt, ammonium diH₂ phosphate, Quartz, Ceramix A & B

$Q = dF$ (charge sensitivity) $\frac{F}{A} = \epsilon \cdot \frac{\Delta t}{t}$ $A = w^2$

$E = \frac{Q}{C}$ $C = \frac{A \epsilon \epsilon_r}{t}$ $\therefore Q = d \cdot \left(\frac{A \epsilon}{t}\right) (\Delta t)$

$E = \frac{d \cdot \left(\frac{A \epsilon}{t}\right) \Delta t}{\frac{A \epsilon \epsilon_r}{t}} = \frac{d \Delta F}{\frac{A \epsilon \epsilon_r}{t}} = \left(\frac{dt}{\epsilon \epsilon_r}\right) \frac{F}{A}$

$\therefore E = g \cdot t \cdot P = \left(\frac{d}{\epsilon \epsilon_r}\right) (t) \left(\frac{F}{A}\right)$ voltage sensitivity
 $g = \left(\frac{E/t}{P}\right) \rightarrow \text{electric field} : g = \frac{\epsilon}{p}$ Vm/N

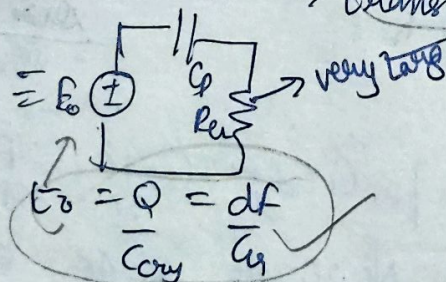
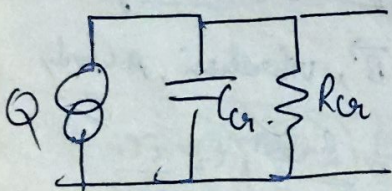
Modulus of elasticity: BaTiO₃ = 12×10^{10} , Quartz = 8.6×10^{10}

$d_{xy} = x$: electrical direction
 y : mechanical

Modes

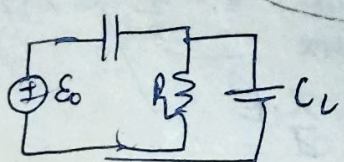
- Thickness shear
- Face shear
- Thickness expansion
- Transverse

Circuit



Thick.
Transverse expansion

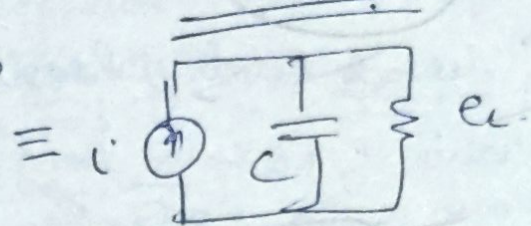
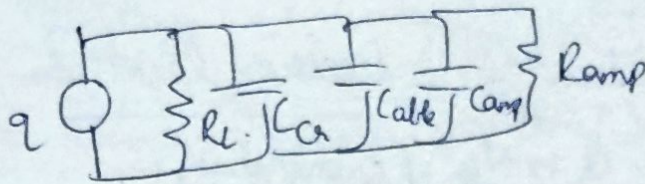
Loading



Here, $R_{cr} \gg R_L$
 $\therefore$ ignore R_{cr}

$$E_L = \frac{\omega C_{cr} R_L}{\sqrt{1 + \omega^2 (C_{cr} + C_c)^2 R_L^2}} E_0 \quad \text{at } \omega \text{ (medium, high)}$$

$$E_c = \frac{E_0}{C_{cr} + C_c}$$



$$C = C_1 + C_2 + C_3$$

$$R = R_{amp} // R_L$$

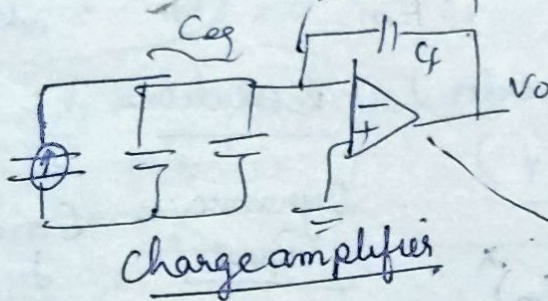
$$i = \frac{dq}{dt} = K \left(\frac{d\pi_1}{dt} \right)$$

$q = k \pi \rightarrow$ displacement sensitivity

$$\frac{E_i}{X_1(s)} = \frac{K \tau s}{1 + \tau s} \quad || = \frac{\omega \tau}{\sqrt{1 + (\omega \tau)^2}}$$

$$K = \frac{e_L}{24^\circ}$$

no static

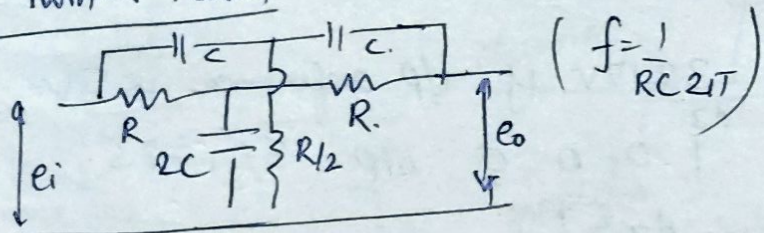


$$\frac{K \tau s}{1 + \tau s}, K = \text{voltage sensitivity} = \frac{K_g}{C_f}$$

$$M \approx \frac{\tau_{io}}{\sqrt{1 + \tau^2 \omega^2}} \quad M \approx 0.95\%$$

Charge amplifier

Twin-T Notch



$$\frac{e_o}{e_i} = -\frac{C_{eq}}{C_f}$$

$$e_i = \frac{q}{C_{eq}} \therefore e_o = -\frac{q}{C_f} = -\left(\frac{d\pi}{C_f} \right)$$

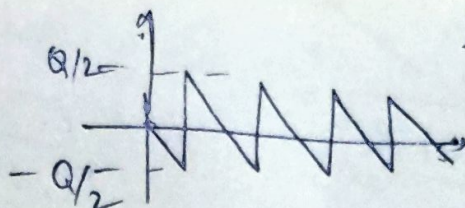
$$g_{33} = \frac{\text{field produced in direction 3}}{\text{applied stress } \tau_{31}}$$

$$d_{32} = \frac{\text{charge in } d_3}{\text{force in } d_2}$$

5/P

any

ADC



$MSB = R$
 $LSB = 2^{n-1} R$

→ 2^n discrete levels, Resolution = $\frac{1}{2^n}$, Decision level = 2^{n-1}

RMS value of $E_q = \frac{Q}{2\sqrt{3}}$
(Q-error or noise)

$Q = \frac{V_{FS}}{2^n}$ (Quantization error or resolution)

→ Aperture time: sample hold circuit
maximum rate of change occurs at instant voltage goes thru '0'

$\therefore \Delta E = \frac{d}{dt}(E_m \sin \omega t) \times t_0 = E_m \omega t_0$

Aperture time = $\frac{\Delta E}{\omega E_m} = t_0 = \frac{\Delta E}{\omega E_m}$ ✓ $\frac{\Delta E}{E_m} = \frac{1}{2^n}$ (Range of convert)

Range of MSB = $\frac{1}{2}$ (range of converter)

LSB = $\frac{1}{2^n}$

Resolution = $\frac{V_{FS}}{2^n}$

Dynamic Range = 6n dB
no. of bits

$f_s = 5 \text{ fm}$ (minimum sampling) = 5 fm

→ Find SAR of 4 bit 3.217V if i/p reference is 5V.

a) Set $d_3 d_2 d_1 d_0$ $\overset{d_3}{1} 0 0 0$ o/p $5/2^1 = 2.5$
 $3.217 > 2.5 \therefore d_3 = 1$

b) Set $d_2 = 1$ o/p $2.5 + 5/2^2 = 3.75 > 3.217$
 $\therefore d_2 = 0$

c) Set $d_1 = 1$ o/p $2.5 + \frac{5}{2^3} = 3.125 < 3.217$

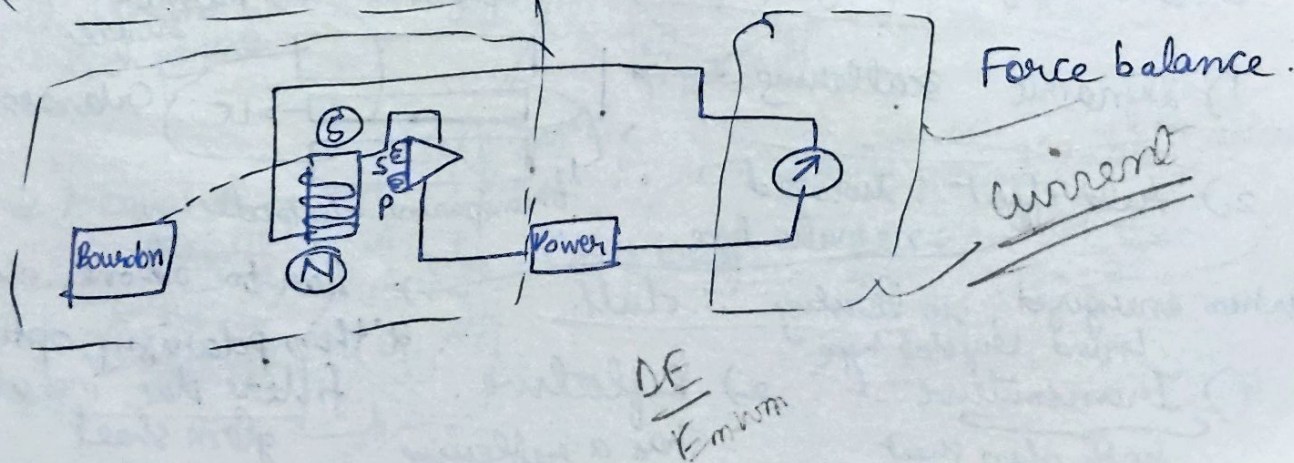
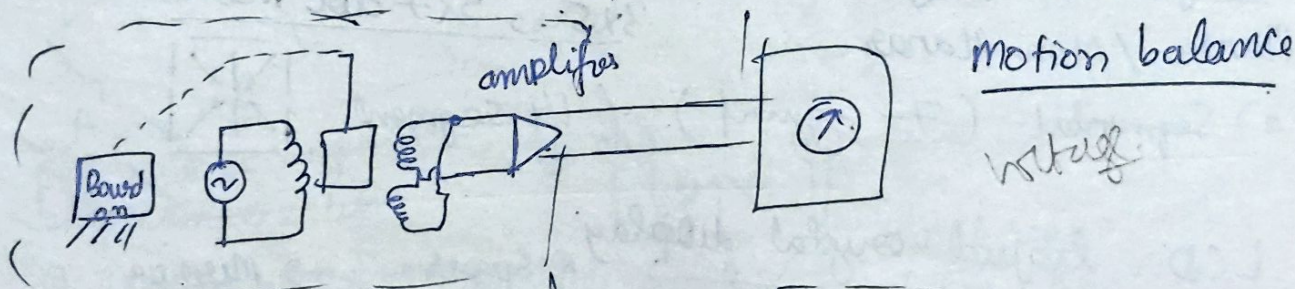
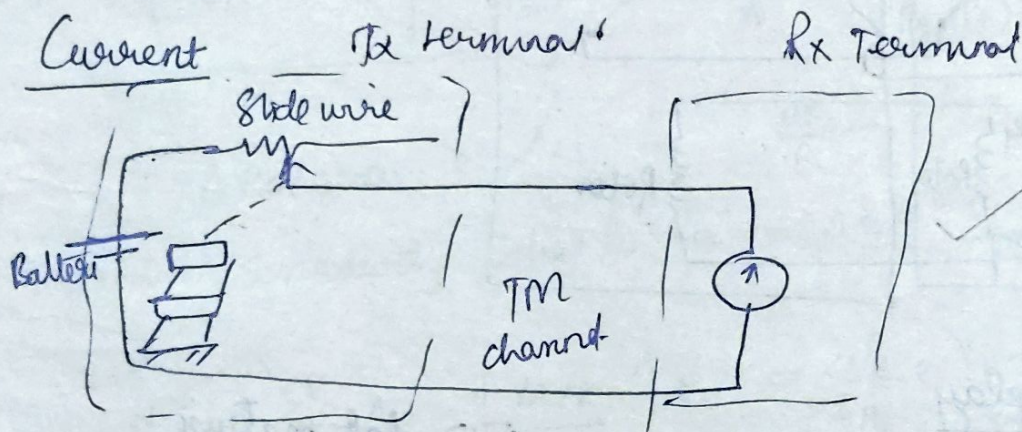
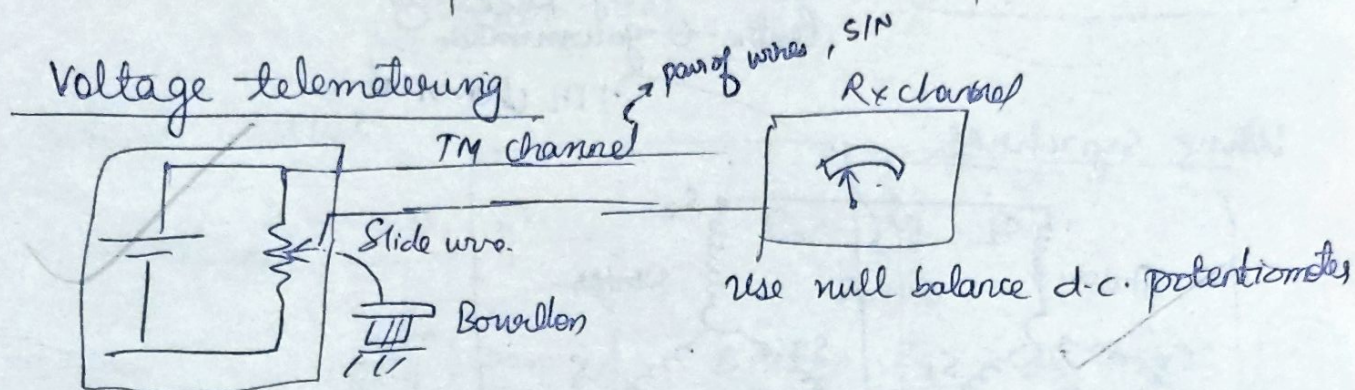
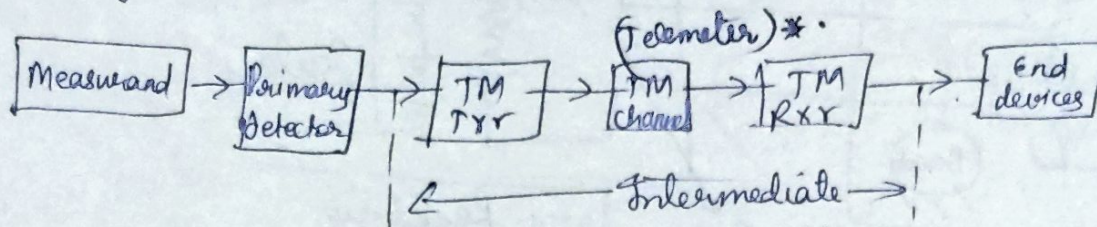
d) Set $d_0 = 0$ o/p $3.125 + \frac{5}{2^4} = 3.4375 < 3.217$
 $d_0 = 0$

$\therefore 1010$

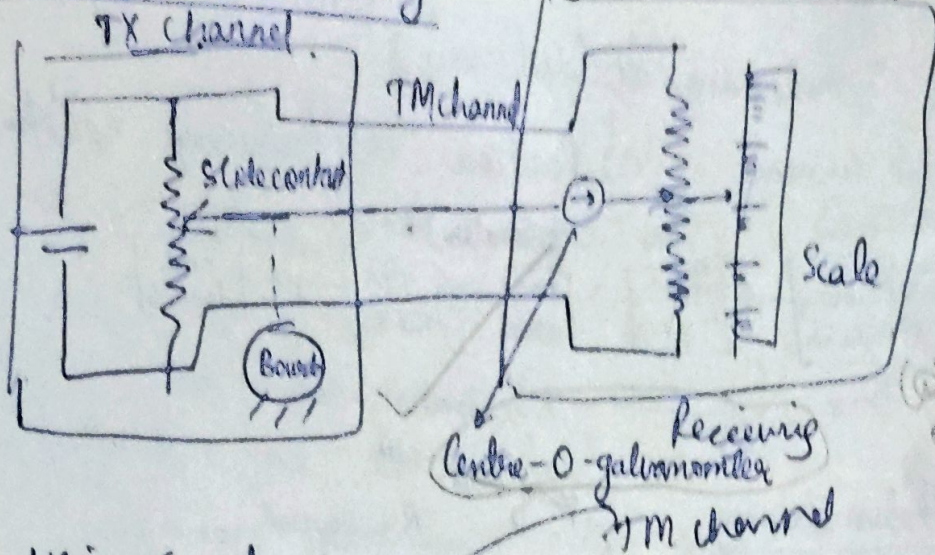
Ramp: $E = \frac{T_2}{T_1} \cdot E_{ref}$
 T_2 = count when i/p voltage applied
 T_1 = u u ref u u

Telemetry : measuring at distance

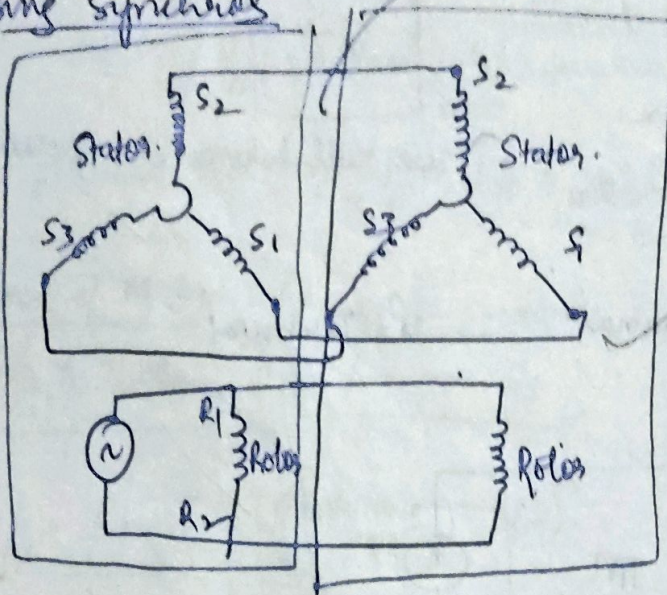
a) Voltage b) Current c) Position d) Frequency e) Puls



Position telemetering



Using Synchros

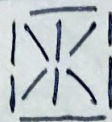


Digital display

Planar / Non Planar

3x5, 5x7 dot matrix

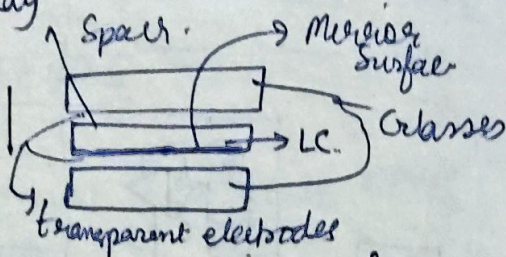
a) Segmental (7-segment) (14 segment)



LCD: Liquid crystal display

1) Dynamic scattering

2) Field effect: twisted nematic type.



When energized, no twisting $\therefore$ dull.
Liquid crystal type.

1) Transmissive
both glass sheet transparent

2) Reflective
has a reflecting

1) as to above, except
2) thin polarizing optical filters place in each glass sheet

74141 (BCD-decimal)

7447 (BCD-7 segment-converter)

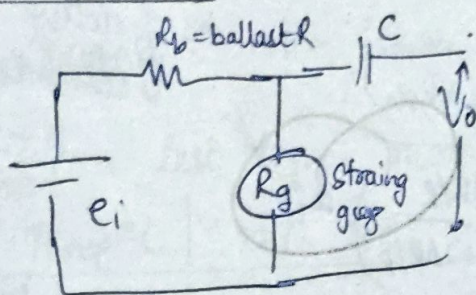
4 1/2 ———— ⊖ ^{10¹²}
3 1/2 ————

$\frac{1}{10^n}$, $n = \text{num. of full digits}$

Sensitivity = $\frac{(\text{full scale})}{\text{min}} \times R_{\text{end}}$

Measurement

ballast circuit



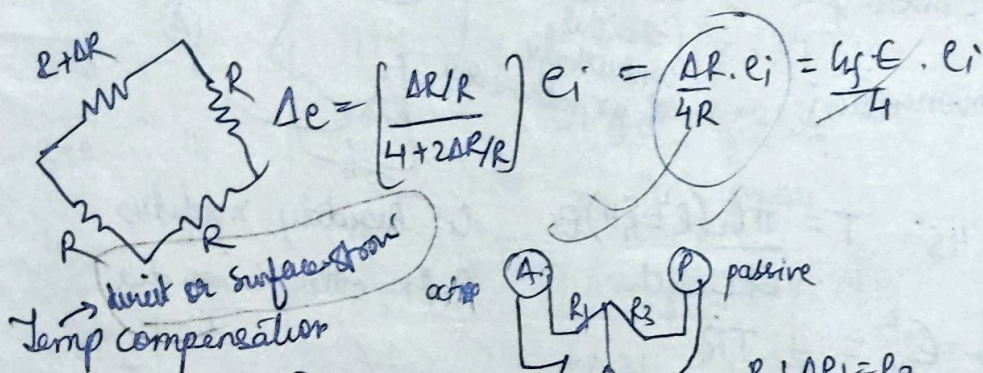
$$e_o = \frac{R_g}{R_g + R_b} \cdot e_i$$

$$\frac{\Delta e_o}{e_o} = \frac{R_g}{(R_g + R_b)^2} \cdot dR_g = \frac{R_b R_g}{(R_g + R_b)^2} \cdot \left(\frac{dR_g}{R_g} \right)$$

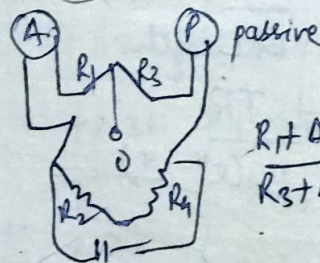
$$\frac{\Delta R}{R} = 4f \cdot e \cdot \epsilon$$

$\therefore \Delta \epsilon$ and strain

Maximum sensitivity when $R_b = R_g \therefore \Delta \epsilon = \frac{G_f \cdot e}{4f} \cdot e_i$

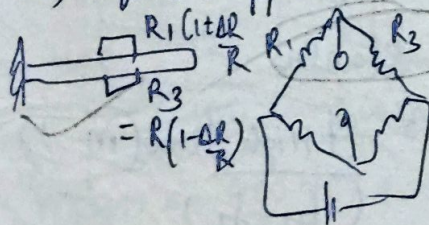


1) Adjacent arm Dummy gauge / compensate



$$\frac{R_1 + \Delta R_1}{R_3 + \Delta R_3} = \frac{R_2}{R_4}$$

2) equal & opposite strains $\therefore$ use 2 active gauges both active



Half bridge

$$\frac{\Delta R}{2R} \cdot e_i = \frac{4f \cdot e \cdot \epsilon}{2}$$

3) 4 active gauge



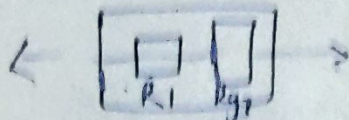
$$\Delta R_i = (1 + \nu) \epsilon_i \frac{R_i}{2}$$

Opposite arms
= similar strain

4) Poisson

Right angles

$$\Delta \epsilon_x = (1 + \nu) \epsilon_y \frac{R}{L}$$



$$\text{effective strain} = \epsilon = \frac{R_g}{\eta G (R_g + R_{sh})}$$

η = no of active elements

$$5) \text{ Load cell } \Delta \epsilon_i = \eta (1 + \nu) \left(\frac{Q R / R}{1 + \nu \Delta R / R} \right) \epsilon_i$$

Roette = three (3) strain gauge oriented in diff. directions
don't know direction of principal stress

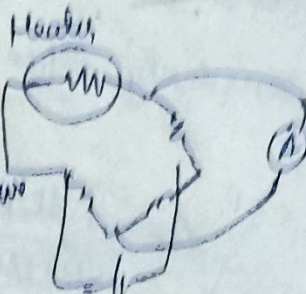
Vacuum Press

1) Thermocouple

2) Pirani

(thermal conductivity of gas with pressure)

3) Ionization



$$\text{Torque } T = \frac{\pi G (R^4 - r^4) \theta}{2 L \text{ length}}$$

G = Rigidity modulus
 R, r = outer, inner dia

$$\text{Strain} = \epsilon_{45}^0 = \pm \frac{TR}{\pi G (R^4 - r^4)}$$

$$\text{Speed} = \frac{\text{Pulses/Second}}{\text{Teeth}}$$

Tach

$$\eta = \frac{f}{m} = \frac{\text{Stator/rev}}{\text{rev/rev}}$$

Stator

$$= \frac{f_{\text{High}} - f_{\text{Low}} (m-1)}{(f_{\text{H}} - f_{\text{L}})}$$

→ 10.03, 10.08, 10.11, 10.10.

1) Avg = 10.08 b) Range of error (max) (Avg)
 $\therefore I_{max} = 10.11 \therefore \text{Range} = 10.11 - 10.08 = 0.03$

($I_{avg} - I_{min}$): minimum Range = $10.08 - 10.03 = 0.05$

$\therefore \text{Avg range of error} = \frac{0.03 + 0.05}{2}$

→ Efficiency = $\frac{F_{full\ scale}}{\text{Power, full scale}}$ ± 0.04 (? max-min)

Accuracy = 100 - error, = $(M - T) / T \times 100\%$

→ In a test temp. measured 100 times.

Temp °C	397	398	399	400	401	402	403	404	405
Freq of occurrence	1	3	12	23	37	16	4	2	2
T	f	T x f	Deviation	f x d	d ²	f x d ²			
397	1	397	-3.78	-3.78					
398	3	1194	-2.78	-8.34					
399	12	4788	-1.78	-21.36					
400	23	9200	+0.22	17.94					
401	37	14837	+1.22	8.14					
402	16	6432	+2.22	19.52					
403	4	1612	2.22	8.88					
404	2	808	3.22	6.44					
405	2	810	4.22	8.44					
	100	40078		$\Sigma 1 = 102.8$					$\Sigma 191.08$

mean = $\frac{40078}{100} = 400.78$

Mean deva = $\frac{102.8}{100} = 1.028$

$$\text{Stand. dev.} = \sqrt{\frac{191.08}{100}} = 1.38$$

$$\text{Prob. error} = 0.6745 \sigma = 0.6745 \times 1.38 = 0.93$$

$$\text{Prob. error of mean} \therefore \frac{91}{\sqrt{100}} = \frac{0.93}{10} = 0.093$$

$$\text{Std. deviation of mean } \sigma_m = \frac{1.38}{\sqrt{100}} = 0.138$$

$$\text{Std. } \sigma_m = \frac{\sigma_m}{\sqrt{2}} = \frac{0.138}{\sqrt{2}} = 0.0796$$

Standards of Measurement

- 1) International
- 2) Primary
NPL
- 3) Secondary
Industry
- 4) Working
Standard
Ref.

$$\text{Statiz error} = (\text{measured} - \text{true})$$

$$\text{Relative S.E.} = \frac{m - t}{t}$$

$$\text{Statiz correction} = S_c = A_t - A_{\text{meas}} = -(\text{Statiz error})$$

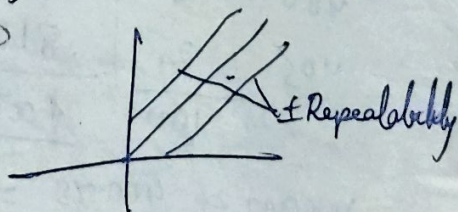
अपेक्षा अपेक्षा
Reproducibility : degree of closeness with which a given value may be repeatedly measured.

perfect reproduction = no drift. (not a function of time)

Repeatability स्टोचिस्टिक.

while reproducibility is specified in terms of scale readings over a given period of time, repeatability is variation of.

Scale values and is random.



$$\text{Inverse sensitivity} = \frac{1}{\text{scale factor}} = \frac{1}{\dots}$$

→ SA ammeter has resistance 0.01Ω . Determine η .

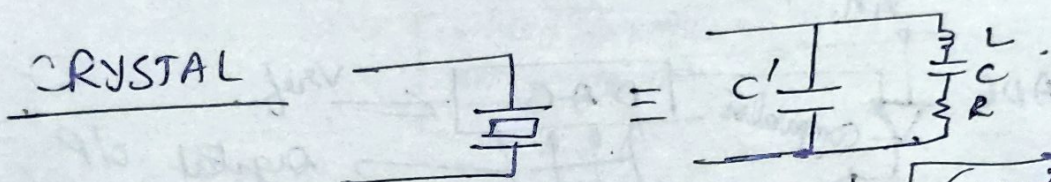
$$\eta \text{ of instrument} = \frac{I_{fs}}{P_{fs}} = \frac{I_{fs}}{I_{fs}^2 R} = \frac{1}{R \cdot I_{fs}} = \frac{1}{0.01 \times 5 \times 100} = 20 \text{ A/W}$$

$$\eta = \frac{E_{fs}}{P_{fs}}$$

$$\text{Linearity} = \frac{\text{maximum resistance/displacement deviation}}{f.s. \text{ deviation}} \times 100$$

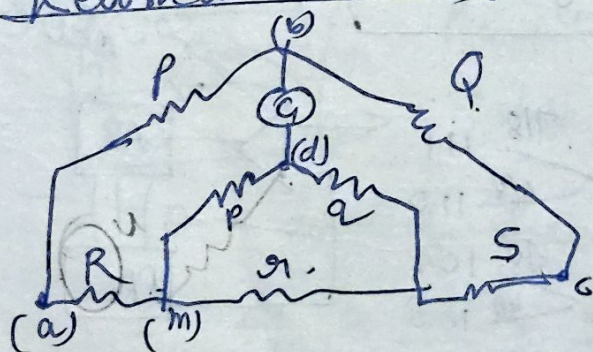
for < 20 observation, we use 's' instead of σ for s.d.

$$V = 8^2$$



Two frequencies $f_1 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$ $f_2 = \frac{1}{2\pi} \sqrt{\frac{(1+C/C')}{LC}}$, $\frac{C}{C'} \ll 1$
 $C' > 7C$

Kelvin double bridge



$$P/Q = p/q$$

$$E_{ab} = E_{amd}$$

$$E_{ab} = \frac{P}{P+Q} \cdot E_{ac} = \frac{P}{P+Q} \cdot I \{ R+S+\frac{p}{q} \}$$

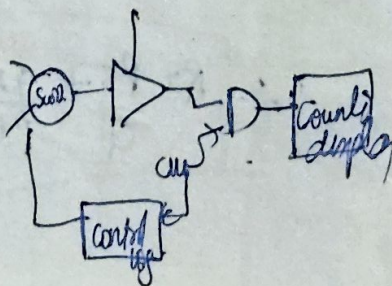
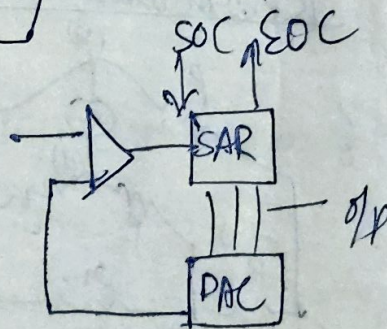
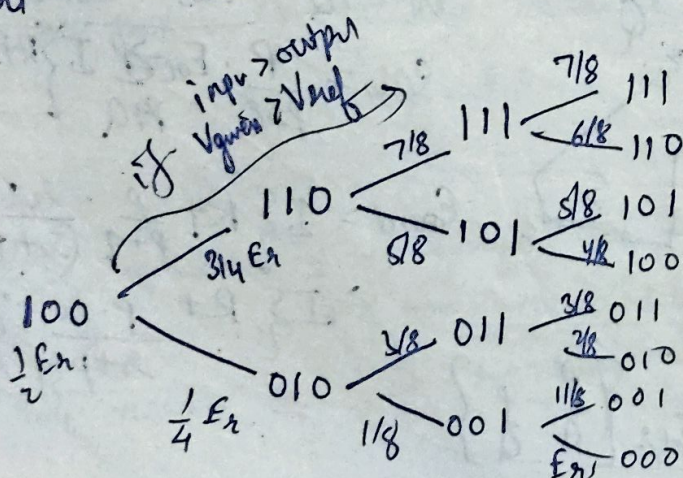
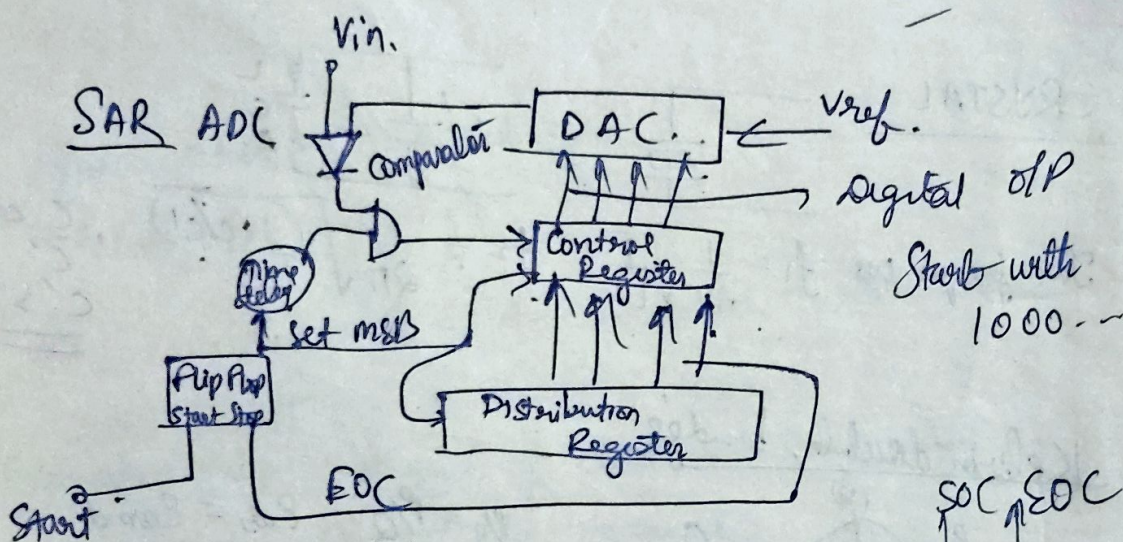
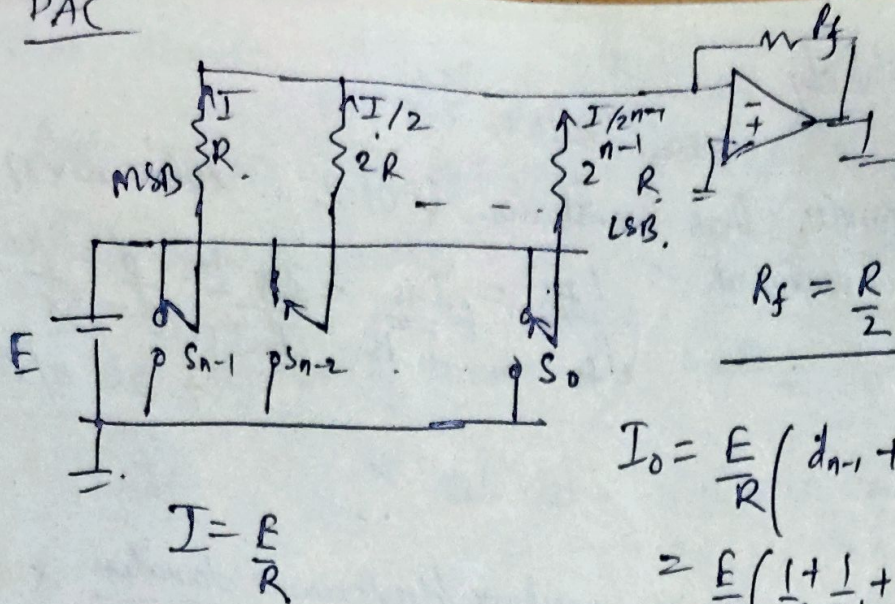
$$E_{amd} = I \{ R + \frac{p}{p+q} \cdot \frac{r}{s} (p+q) \}$$

$$= I \{ R + \frac{p \cdot r}{s+p+q} \}$$

$$\therefore R = \frac{pS}{Q} + \frac{qr}{p+q+s} \left\{ \frac{p}{q} - \frac{p}{q} \right\}$$

$$I \left(R + \frac{p}{p+q+s} \cdot \frac{r(p+q)}{p+q+s} \right)$$

DAC

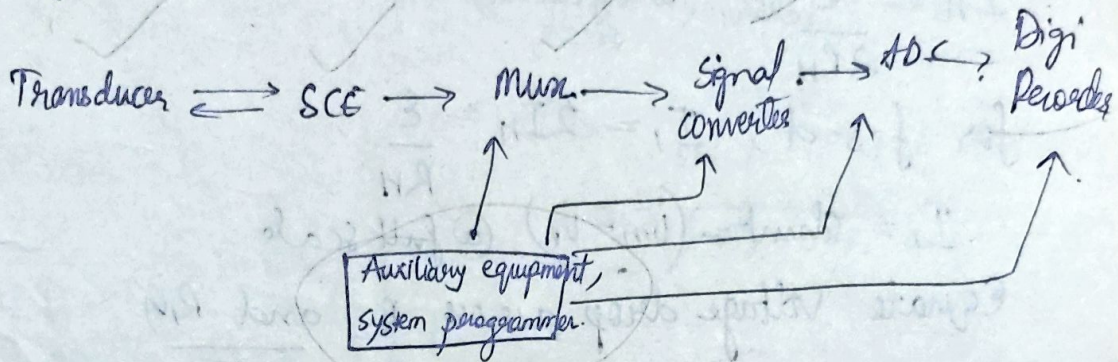


DATA ACQUISITION : Basics

Analog DAS

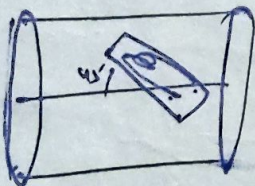
- 1) Transducer
- 2) SCE
- 3) Calibrating equipment
- 4) $\int$ equipment
- 5) Visual display device
- 6) Analog recorder
- 7) Analog Computers
- 8) High Speed Camera/TV

Digital DAS



Analog : wide freq. width / low accuracy be tolerated
 Digital : narrow bandwidth, high

Q →



$R_{sa} = 120 \text{ R}$, $GF = 2$, $G_{shear} = 80$ modulus
 $r_{shaft} = 15 \text{ mm}$, $\Delta R = 0.24$. Find Torq

$$\theta = \text{angle of shear} = \frac{2T}{\pi R^3 G}$$

$$\epsilon = \frac{\Delta L}{L} = \frac{\theta/2}{\sqrt{2}} = \frac{\theta}{2} \quad \therefore \frac{\Delta R}{R} = GF \cdot \epsilon = \theta = \frac{0.24}{120}$$

$$\therefore T = \frac{\pi R^3 G \theta}{2} = 848 \text{ Nm.}$$

$$= 2 \times 10^{-3}$$

Ohmmeter

① Series

@ AB: short circuit

adjust R_2
 $I_{\text{meter}} = I_{\text{full scale}} \rightarrow R = 0$

@ AB = ∞ , $i_m = 0$

@ $R_{AB} = R_H$, half scale, $I_{\text{meter}} = \frac{I_{\text{full scale}}}{2}$

$$\therefore R_H = R_{H2} \therefore R_H = \frac{R_1 + R_2 R_m}{R_2 + R_m} \quad \text{--- (1)}$$

$$I_H = \frac{E_{\text{battery}}}{2R_H} \quad (\text{battery current})$$

$$\text{for f.s.d, } I_1 = 2I_H = \frac{E}{R_H}$$

$$I_2 = \text{shunt} = (i_m - I_1) \text{ @ full scale}$$

Equate voltage drop across R_2 and R_m

② Shunt

$R_{AB} = \text{short}$, $i_m = 0$

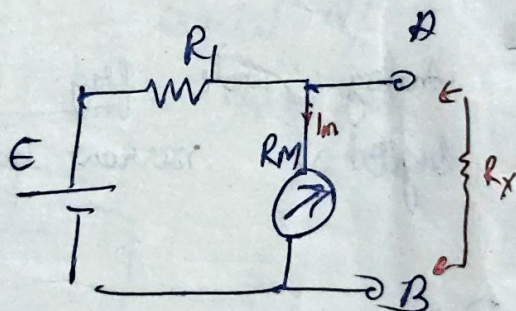
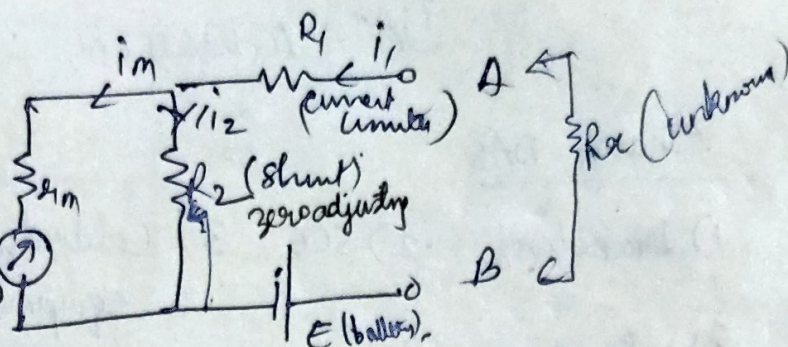
$R_{AB} = \infty$, $i_m = I_{\text{f.s.d}} = \frac{E}{R_1 + R_m}$

@ half scale, $i_m = \frac{1}{2} I_{\text{fs}}$, $R_x = R_H$

$$\text{@ any } R_x, I_m = \left(\frac{E}{R_1 + \frac{R_m R_x}{R_m + R_x}} \right) \left(\frac{R_m}{R_m + R_x} \right)$$

$$S = \frac{I_m}{I_{\text{fs}}}$$

$$R_H = \frac{R_1 R_m}{R_1 + R_m} \quad (R_1 \parallel R_m)$$



Errors

- 1) Gross : Human mistakes
- 2) Systematic : Enviro / Instru / Observational
- 3) Random (Residual) due to chance.

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} dx = 1.$$

$$\begin{aligned} V_{x1} &= \left(\frac{\partial x}{\partial x_1} \right)^2 V_{x1} \quad \sigma_x = \sqrt{\left(\frac{\partial x}{\partial x_1} \right)^2 V_{x1} + \left(\frac{\partial x}{\partial x_2} \right)^2 V_{x2} + \dots} \\ &= \sqrt{V_{x1}} = \sqrt{(\)^2 \sigma_{x1}^2 + (\)^2 \sigma_{x2}^2 + \dots} \\ \therefore \sigma &= \sqrt{\sigma_{x1}^2 + \sigma_{x2}^2 + \dots}; \sigma_{x1} = \left(\frac{\partial x}{\partial x_1} \right)^2 \sigma_{x1} \end{aligned}$$

Thermocouple

T: copper constantan

J: Iron-constantan.

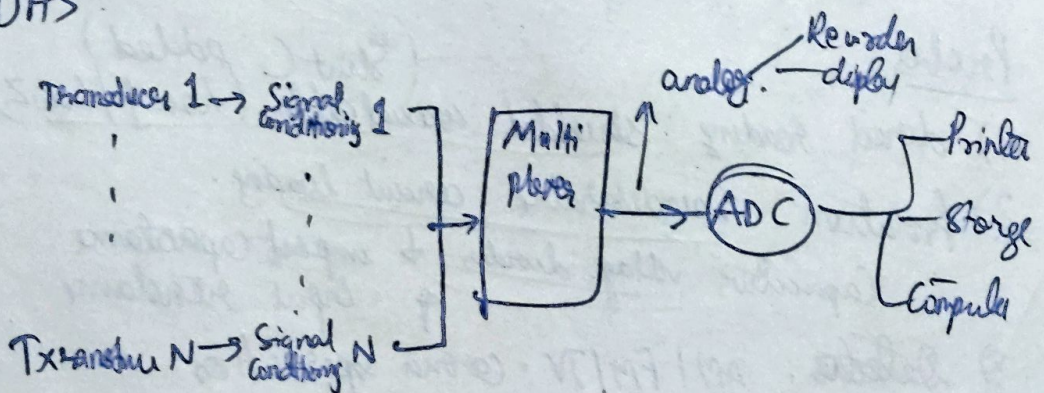
K: chromel alumel

E: chromel constantan.

S: Pt-Pt-Rh

(10) (R=13)

DAS



Some extra CRO

Most CRO beam deflection circuits require ramp voltage.

Circuits for ramp voltage: time base/sweep generator

① Free running ② Triggered ③ Synchronised sweep ④ Non sawtooth

→ Synchronised with measured signal (submultiple freq)

→ Blanking: sawtooth sweep voltage

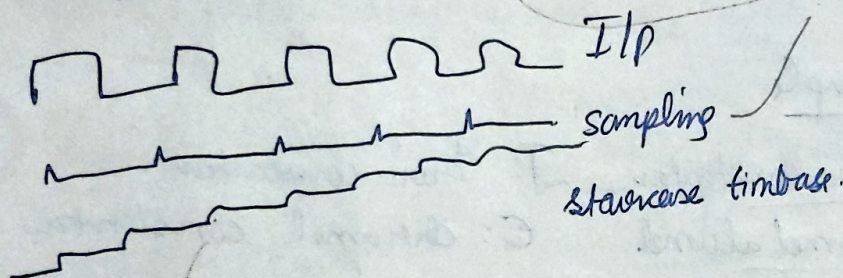
↳ slow: solid line fast: thin/dim line

retrace/flyback time

↳ has to be blanked out using high negative voltage

Sampling type: Normal oscilloscope in 50-300 MHz range.

above this, need Sampling type



Storage: variable persistence = matching sweep time.

Storage tube (collection mesh + storage target)

Probes

1) direct reading: shielded coaxial cable (short C added) (low f / low Z)

2) Isolation: undesirable circuit loading.

Capacitive voltage divider ↓ input capacitance
↑ input resistance

3) Selectors: AM / FM / TV. common operation

$$F = eE_0 = 1 \text{ mV}^2 \quad v = \sqrt{\frac{2eE_0}{m}}$$

$$F = \frac{eE_0}{m} = a$$

$$y = \frac{1}{2} \left(\frac{eE_0}{m} \right) t^2$$

$$B \times \frac{L^2}{2} = \frac{1}{2} \left(\frac{eE_0}{m} \right) t^2$$

$$t = \frac{x}{v} \quad \frac{dy}{dx} = \left(\frac{eE_0}{m \cdot v} \right) x + \frac{1}{2} \frac{m \cdot v}{eE_0} = \frac{1}{2}$$

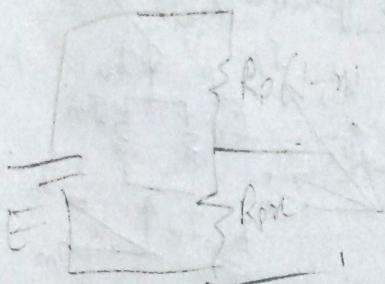
$$P = L I_d B \sqrt{\frac{q}{2mV_0}}$$

Sensitivity

$$S = \frac{D}{E_d}$$

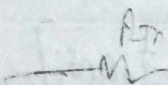
$$S = \frac{L I_d}{2 e E_d B}$$

diff. fail



$$\frac{R_P}{R_P + R_P(1-x)} = \frac{R_P}{R_P(1+x)}$$

$$R_P(1+x)$$



$$V = \frac{x R_L E}{R_L + R_P(x)(1-x)} = \frac{x E}{1 + R_P(x)(1-x) \frac{1}{R_L}}$$

$$L I_d B \sqrt{\frac{q}{2mV_0}}$$

$$N \cdot L = E - V$$

$$= x E \left(1 - \frac{1}{1 + R_P(x)(1-x) \frac{1}{R_L}} \right) = \frac{x E}{1 + R_P(x)(1-x) \frac{1}{R_L}}$$

$$L I_d B \sqrt{\frac{q}{2mV_0}}$$

$$\frac{R_P x^2 (1-x) E}{R_L}$$

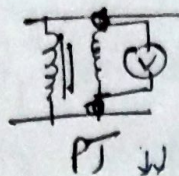
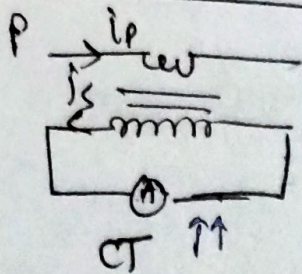
$$\frac{1 + R_P x (1-x) \frac{1}{R_L}}{R_L}$$

$$R_L > R_P$$

$$R_P < R_L$$

$$x^2 - x^3 = \frac{R_P}{2 R_L} x - 3x$$

Instrument Transformers



$$R = \text{transfo}^n \text{ ratio} = \frac{\text{Primary phase}}{\text{Secondary phase}}$$

$$\text{Nominal Ratio} = \frac{\text{Rated } I_s}{\text{Rated } I_p} = \frac{\text{Rated } V_p}{\text{Rated } V_s}$$

$$\text{Turns Ratio} = \frac{N_p}{N_s} = \frac{I_s}{I_p} \text{ (CT)}, \frac{V_p}{V_s} \text{ (PT)}$$

$$\text{Correction factor} = \frac{R}{\text{Nominal Ratio}}$$

CT

$N_p, N_s, \theta = \text{phase angle of } X_{fer}$

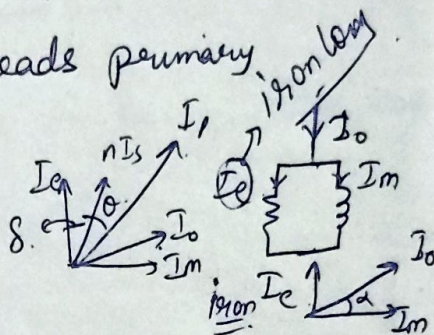
$$\delta = \text{Secondary voltage } \{ \text{current} \} = \tan^{-1} \left(\frac{x_s + x_e}{r_s + r_e} \right)$$

$$R = n + \frac{I_0}{I_s} \sin(\delta + \alpha), \quad I_0 = \text{magnetizing current} \quad [\alpha - \text{small}]$$

$\theta = \text{phase angle} = +ve \text{ if secondary leads primary}$

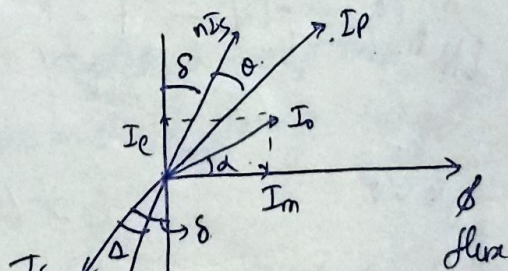
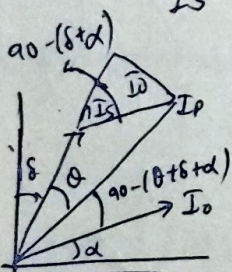
$$= \frac{I_0 \cos(\alpha + \delta)}{n I_s + I_0 \sin(\alpha + \delta)}, \quad n I_s \gg I_0$$

$$\approx \frac{I_0 \cos(\delta + \alpha)}{n I_s} \quad (\alpha - \text{small})$$



$$\text{Ratio error} = \frac{(n - R) \times 100\%}{R}, \quad \text{Phase angle error} = \frac{180}{\pi} \left[\frac{I_m \cos \delta - I_e \sin \delta}{n I_s} \right]$$

$$R \approx n + \frac{I_0}{I_s}, \quad \theta \approx \frac{180}{\pi} \frac{I_m}{n I_s} \quad (\delta \text{ small})$$



$$\text{Ratio} = n + \frac{I_0}{I_s} \sin(\delta + \alpha)$$

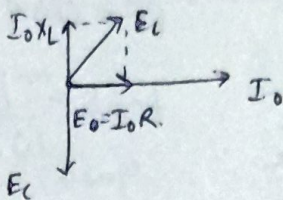
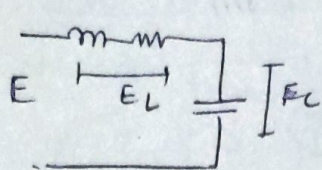
$$PA: \frac{180}{\pi} \left[\frac{I_0 \cos(\alpha + \delta)}{n I_s} \right]$$

$$\text{Error in energy meter due to lag adjustment} = \frac{\sin(\Delta - \phi) - \cos \phi}{\cos \phi}, \quad \Delta = \text{lag adj}$$

Q-meters

$$Q = \frac{\omega_0 L}{R}$$

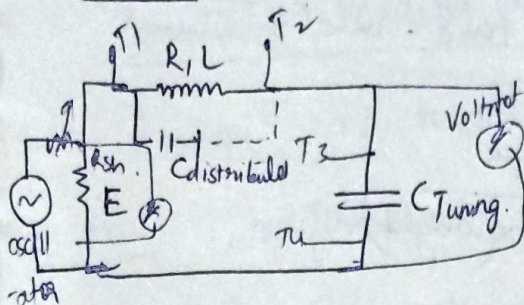
$R \rightarrow \infty$, X different (skin eff)



$$E = I_0 R \Rightarrow \frac{E_0}{E} = \frac{I_0 X_C}{I_0 R} = \frac{\omega_0 L}{R} = Q$$

$\Rightarrow$ output across C

Practical



vary C $\rightarrow$ bring resonance
 f_1, f_2

$$Q_{meas} = \frac{\omega_0 L}{R + R_{sh.}} \quad C_{distributed}$$

$$Q_{tune} = \frac{\omega_0 L}{R} \quad C_{tuning}$$

$$\left. \begin{matrix} Q_{meas} \\ Q_{tune} \end{matrix} \right\} = \frac{1}{\omega C(R + R_{sh.})}$$

also, $Q_{tune} = Q_{meas} \left(1 + \frac{C_d}{C} \right)$ $C_1, C_2 \rightarrow 2$ resonance.

$$f_1 = \frac{1}{2\pi \sqrt{L(C_1 + C_d)}} \quad C_1 = C_{tuning}, \quad f_2 = 2f_1 = \frac{1}{2\pi \sqrt{L(C_2 + C_d)}}$$

Distortion analysis

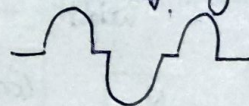
Types of distortion 1) Freq D : amplification factor = $f(f_{req})$

2) Phase D : L, C shift

3) Amplitude : harmonics

4) intermodulation : heterodyning

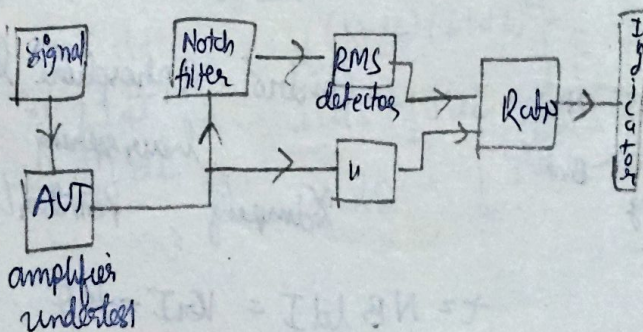
5) cross over di



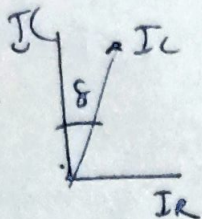
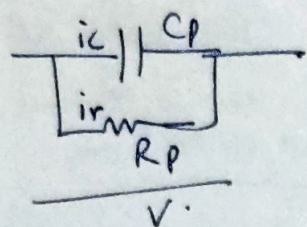
$$THD = \frac{\sqrt{E_2^2 + E_3^2 + \dots}}{E_1}$$

$$= \frac{\sqrt{D_2^2 + D_3^2 + \dots}}{D_1}$$

if noise $THD = \frac{\sqrt{H^2 + N^2}}{\sqrt{F^2 + H^2 + N^2}}$



Loss angle



$$PF = \cos \phi = \sin \delta$$

$$\tan \delta = \frac{1}{\omega \tau}$$

$$Power\ loss = VI \cos \phi = VI \sin \delta$$

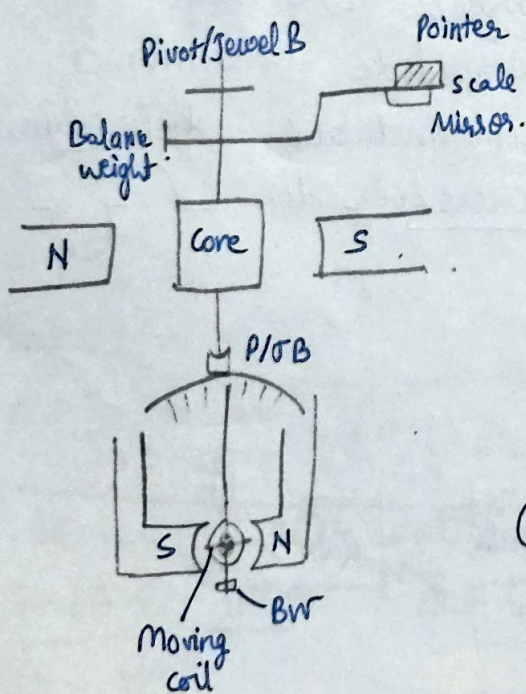
$$\tan \delta = \omega C_s R_s = \omega \tau$$

$\tau / \text{weight ratio} \rightarrow \frac{\text{deflection torque}}{\text{frictional torque} \propto (\text{weight})}$
affects

Damping

- 1) Air friction
- 2) Fluid friction (oil)
- 3) eddy current
- 4) Electromagnetic.

PMMC



moving coil

Copper wire,
mount of Alumi former.

B-system: ALNICO, ALCOMAX

angular span $\sim 90^\circ$

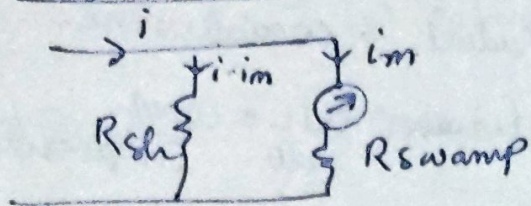
use concentric for higher span

Control: 2 phosphor bronze hairspring

Damping, Pointer/scale

$$\tau = N B l d I = K \phi I = K \theta$$

Effect of Temp change Ammeter



$$R_{swamp} = 30 \times R_{coil}$$

Sensitivity, loading effects

- 1) Error a) magnet weak b) spring weak c) $R \rightarrow \infty$

Adv: i) uniform scale ii) $R_{cons} \downarrow$ iii) high $\tau/mg \uparrow$

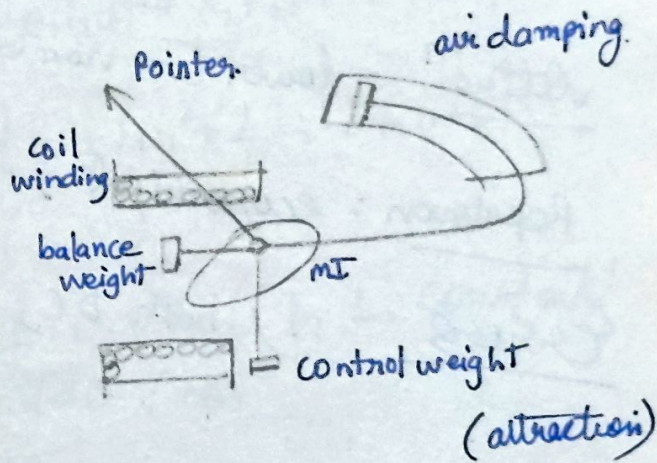
- iv) use shunt etc $\rightarrow$ multirange v) stray B error less
vi) shielded magnet self $\rightarrow$ useful in aircraft/aero

Disadv: i) dc 2) cost

MOVING IRON

high permeability moves in B_{field} of stationary coil (this thing excited by V/I to be measured)

$\rightarrow$ position of minimum reluctance or increase inductance



Electrical energy Supplied = $\uparrow$ in stored + mech. work done.

$$E I dt = \frac{1}{2} (L + dL) (I + dI)^2 - \frac{1}{2} L I^2 + T d\theta$$

$$\Rightarrow L I dI = I L dI + \frac{1}{2} I^2 dL - \frac{1}{2} I^2 dL + T d\theta$$

$$\Rightarrow T d\theta = \frac{1}{2} I^2 dL \Rightarrow T = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

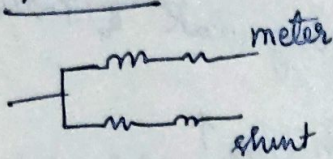
a) attraction type

b) repulsion type : 2 vanes
fixed + movable.
radial + coaxial vane.

Scale: non uniform

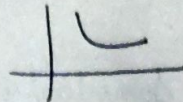
if uniform: $\frac{dL}{d\theta} = \frac{kQ}{I^2}$ or $(I = \text{const})$ linear $\Rightarrow \frac{dL}{d\theta} = \text{const}$ - (impossible)

Shunts

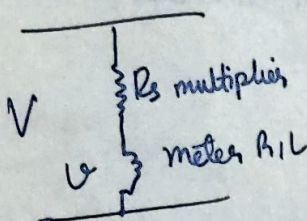


$\frac{I_{sh}}{I_m}$ will be independent of frequency

$$\tau_{\text{meter}} = \tau_{\text{shunt}}$$



Multiplier



$$V = I_m \sqrt{R^2 + \omega^2 L^2} \quad V = I_m \sqrt{(R + R_s)^2 + \omega^2 L^2}$$

$$m = \frac{V}{V} = \sqrt{\frac{(R + R_s)^2 + \omega^2 L^2}{R^2 + \omega^2 L^2}}$$

Attraction : lower L vis a vis repulsion : accuracy over wider range + shunt usage $\uparrow$.

Repulsion : economical, uniform nearby

Errors

i) both DC, AC a) hysteresis $\rightarrow$ make iron parts small.

b) temp : spring, self heating
manganin + $R_s > R_{\text{coil}}$

c) stray B field : shield

assumpⁿ $R_s > R$, $\omega L \ll R_s$
2) AC only

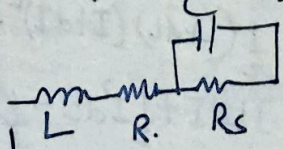
$$i) |Z| = R_s = \left| j\omega L + \frac{R_s}{j\omega R_s C + 1} \right|$$

$$= \left| j\omega L + R_s (1 - j\omega R_s C) \right|$$

$$L = 2.41 R_s C$$

a) frequency. Reactance

$$I = \frac{V}{\sqrt{(R + R_s)^2 + \omega^2 L^2}}$$

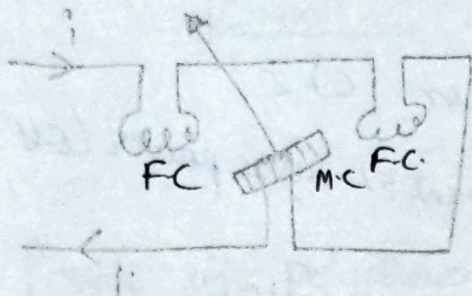


$$C = 0.41 \frac{L}{R_s^2}$$

b) Eddy current

- adv i) universal AC+DC 2) less friction errors. (TWT)
 3) cheap 4) robust 5) accur 6) scale
 7) reversible 8) nonlinear 9) DC-AC diff

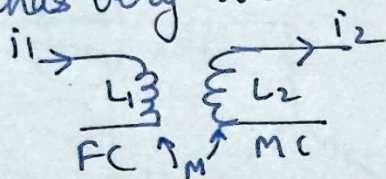
Electrodynamometer : transfer-instrument : Calibrate DC, use with AC.



F.C. : $\div 2$ to give uniform field
 heavy wire + stranded

M.C. , air damping

has very weak shielding $\Rightarrow$ enclosure



flux in F.C. = $\psi_1 = L_1 i_1 + M i_2$ $\psi_2 = L_2 i_2 + M i_1$

i/p energy = $\int e_1 i_1 dt + \int e_2 i_2 dt = \int i_1 d\psi_1 + \int i_2 d\psi_2$

$$\text{energy stored} = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M$$

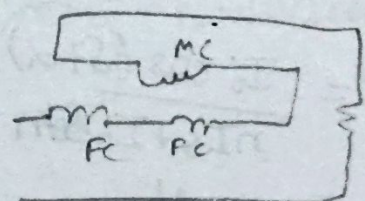
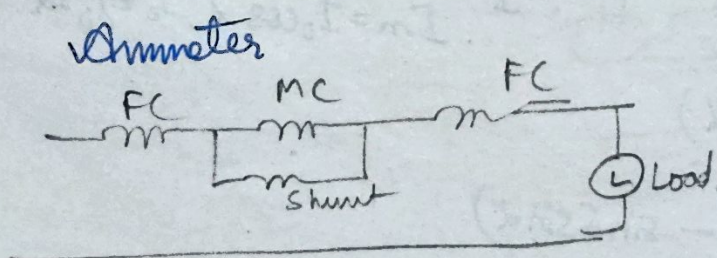
$$\therefore \Delta E = d()$$

$$\text{mech energy} = \text{i/p} - \text{stored} = \int i_1 i_2 dM \quad (L_1, L_2 \text{ constant})$$

$$= \int \tau d\theta \quad (\cos)$$

$$\tau = i_1 i_2 \frac{dM}{d\theta} \quad (\text{torque}) = k\theta$$

$$\theta = \frac{i_{1\text{rms}} i_{2\text{rms}}}{k} \cos\phi \frac{dM}{d\theta}$$



Errors

- 1) low τ/ω 2) freq 3) eddy.

$$\tau = \tau_2$$

- 4) Best $\rightarrow$ use "Astatic system" 5) Temp change.

adv a) no hyst / eddy. air cored

b) precision - grade accuracy for 40-100 Hz

c) transfer: AC + D.C.

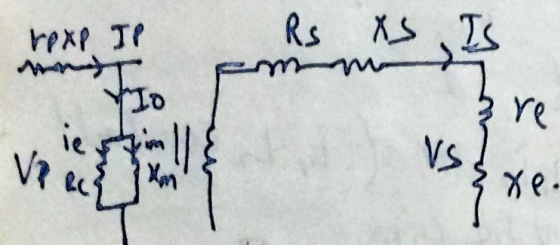
disadv a) low τ/ω b) friction loss c) $\neq$

d) nonuniform. e) high current $\Rightarrow$ more power loss

Half wave Rectifier $\therefore \text{avg} = \frac{1}{\pi} V_m = \frac{\sqrt{2}}{\pi} V_p = 0.45 V_p$

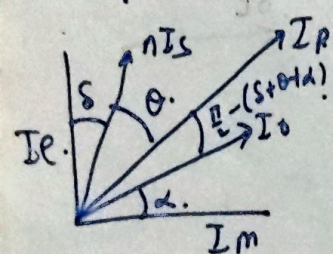
Full wave $V_{av} = \frac{2}{\pi} V_p = 0.9 V_p = \frac{2}{\pi} V_m$

CT



$$\delta = \tan^{-1} \frac{X_s + X_e}{R_s + R_e}$$

$$\alpha = I_0, I_P$$



$$I_P = \sqrt{n^2 I_s^2 + I_0^2 + 2n I_s I_0 \sin(\delta + \alpha)}$$

$$R = \frac{I_P}{I_s} = \frac{n I_s + I_0 \sin(\delta + \alpha)}{I_s}$$

$$= n + \frac{I_0}{I_s} \sin(\delta + \alpha)$$

$$I_m = I_0 \cos \alpha, I_e = I_0 \sin \alpha$$

$$\tan \theta = \frac{I_0 \cos(\delta + \alpha)}{n I_s + I_0 \sin(\delta + \alpha)}$$

θ small, I_0 small.

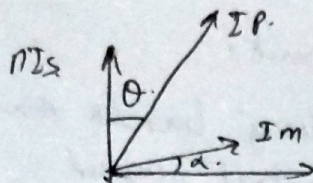
$$\theta = \frac{I_0}{n I_s} (\cos \delta \cos \alpha - \sin \delta \sin \alpha)$$

change

Phase angle = $\frac{I_0 \cos(\delta + \alpha)}{nI_s + I_0 \sin(\delta + \alpha)}$ $\approx \tan \theta \sim \theta$
 (Is rev, Ip)
 ignore.

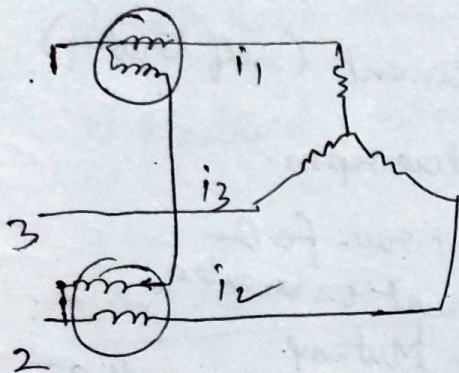
Ratio error : $\frac{I_n - R}{R}$

PL err = $\theta = \frac{I_m \cos \delta - I_e \sin \delta}{nI_s}$

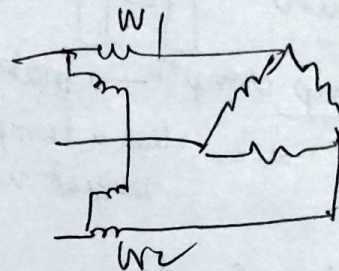


θ = Phase angle.

Power in polyphase system



Star-Y $P_1 = i_1(V_1 - V_3)$ $P_2 = i_2(V_2 - V_3)$
 $P_1 + P_2 = V_1 i_1 + V_2 i_2 + V_3 i_3$

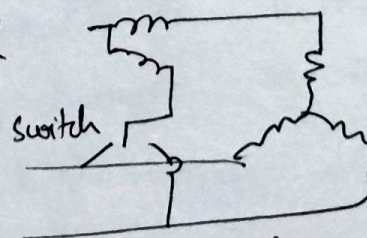


Δ $P_1 = -V_3(i_1 - i_3)$
 $P_2 = V_2(i_2 - i_1)$
 $P_1 + P_2 = V_1 i_1 + V_2 i_2 + V_3 i_3$

instantaneous

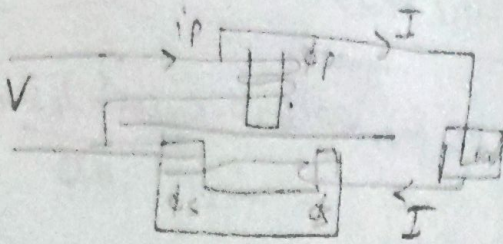
$P_1 = \sqrt{3}VI \cos(30 - \phi)$
 $P_2 = \sqrt{3}VI \cos(30 + \phi)$

1 wattmeter : load balanced



$P_1 = \sqrt{3}VI \cos(30 - \phi)$
 $P_2 = \sqrt{3}VI \cos(30 + \phi)$

Energy meter



$$N = KV I \sin(\Delta - \phi)$$

ϕ = load phase $\angle$.

Δ = phase angle b/w supply voltage, p.c. flux

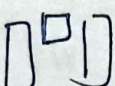
Thus, Δ needs to be made 90 (Lag adjustment)

- 1) adjustable resistance
- 2) shading bands on central magnet

Light loads : friction compensating shading loop between centre pole / disc

Creep : no current flow : friction compensation
drill holes diametrically

Overload : revolve $\rightarrow$ braking $\tau \rightarrow$ eddy current (self braking)

magnetic shunt  + low rpm.

Voltage compens + Temp compens $\rightarrow$ makes it run fast
use a temp shunt on brake magnet with mutemp

$\rightarrow$ decrease in μ with $\uparrow T$

Phase - Freq

Lag adjustment = Magnetic shunt

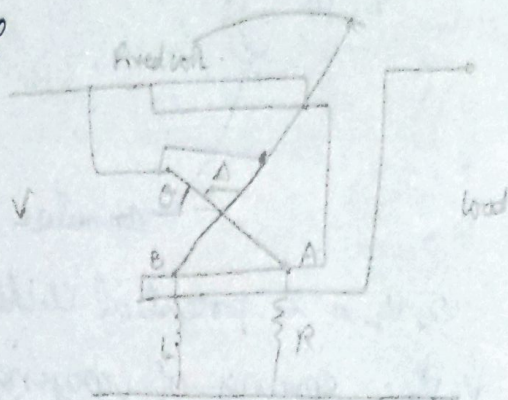
I-F meter

1) Electro-dynamometer

$$\tau_A = KVI M_{max} \cos \phi \sin \theta$$

$$\tau_B = KVI M_{max} \sin \phi \cos \theta$$

$$\tau_A = \tau_B \Rightarrow \theta = \phi$$



Frequency meter

1) Mech. Resonance $\rightarrow$ vibrating reed type.

2) Electrical resonance: Ferrodynamic type
Electrodynamical

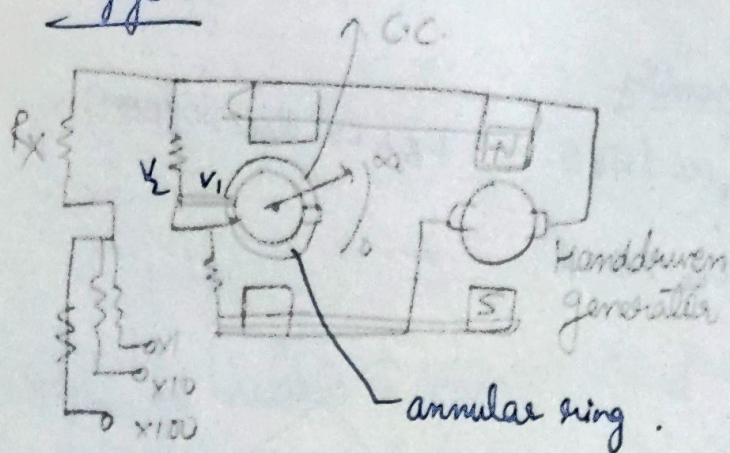
low resistance (Kelvin Double bridge)

- 1) lead
- 2) thermoelectric/Parasite emf
- 3) temp effects
- 4) Contact resistance

High Resist :

- i) insulation R $\sim$ comparable to actual value $\rightarrow$ leakage.
- ii) electrostatic $\rightarrow$ stray charges
- iii) earth
- iv) Cap action
- v) Temp. changes
- vi) high voltage instruments.
- 1) Direct deflection
- 2) loss of charge (1/R C)
- 3) Megohm bridge

Megger



$V_1, V_2 = 2$ potential coils

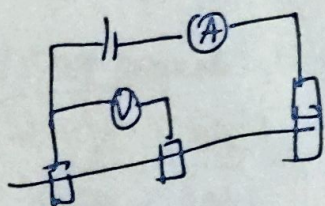
V_1, V_2 : spring of varying stiffness $\rightarrow$ stiff near low, opening up near high

Measuring earth resistance

shape of earth electrode, depth + soil properties

1) Fall of potential

2) Earth tester / megger



Bridge measurement

Error 1) stray conductance 2) stray cap 3) M 4) resistance

Precaution 1) high quality component

2) lay out $\rightarrow$ close together

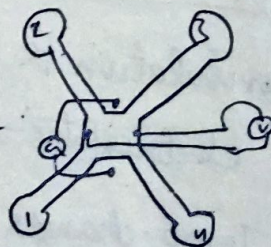
3) sensitivity $\uparrow$

4) stray conductance: insulating steel

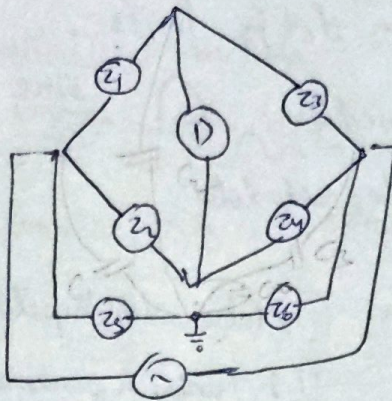
5) eddy current

6) residual error

7) freq, waveform $\Rightarrow$ use filters

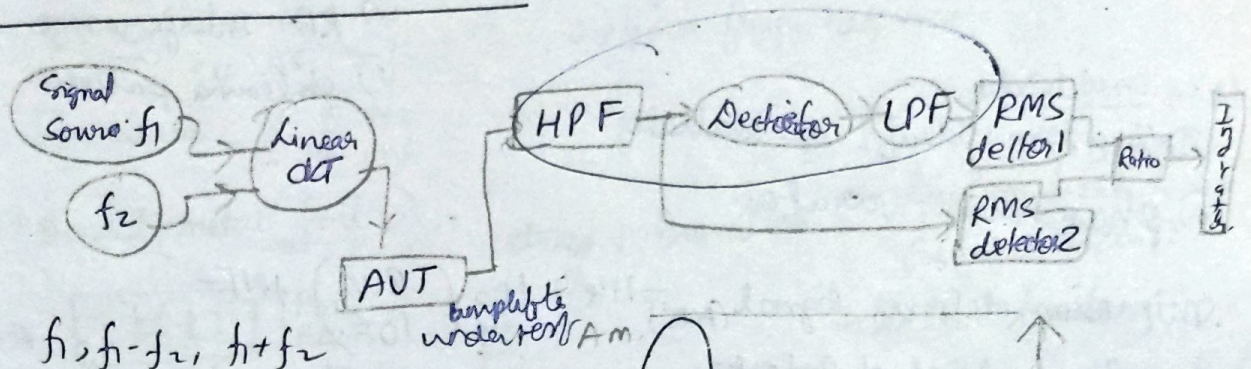


Wagner → remove all earth capacitance



$Z_5/Z_6 = \text{variable impedance of wagner earth}$
 $Z_5/Z_6 = \text{balance bridge with } Z_1/Z_3 \text{ or } Z_2/Z_4$

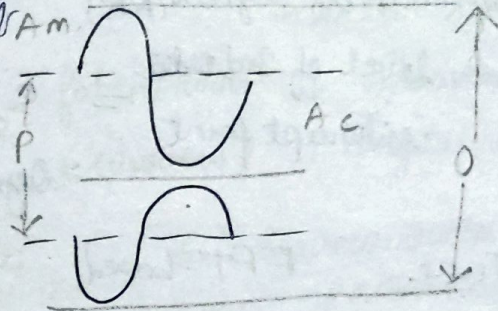
Intermodulation distortion



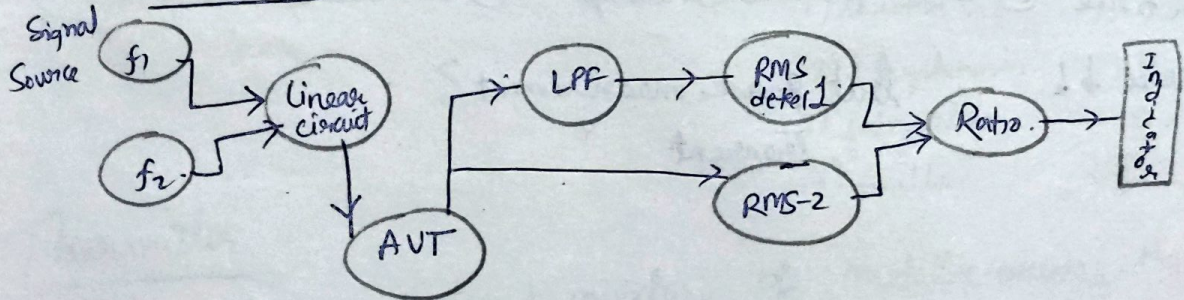
$f_1, f_1 - f_2, f_1 + f_2$

$$\text{IMD} = \frac{Q-P}{P}$$

$$= \frac{A_m}{A_c}$$

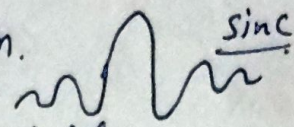


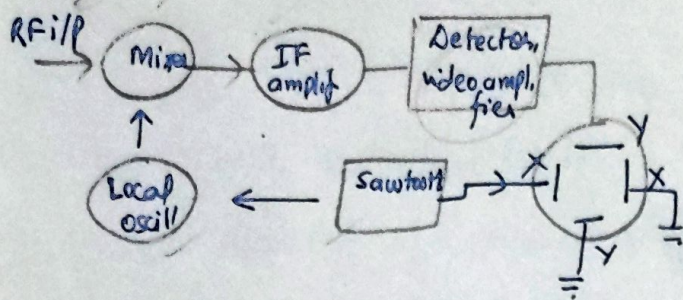
Difference freq. distortion modes



Spectrum Analyser

Continuous wave = f_c , Ampli Modⁿ = $f_c \pm f_s, f_c - f_s$

Phase/Freq modⁿ: $f_c \pm n f_s$. Pulse modⁿ 
 main lobe, side lobe.



i) Pulse width, repetitive rate

ii) Tuning a parametric amplifier

iii) FM deviation

iv) RF interference

v) Antenna pattern

freq instability

① Long term: drift of oscillator

② phase noise: variation

minimum detector signal (MDS) $-114 + \log\left(\frac{BW}{10^6}\right) + NF$

Power level of 3rd order intercept point = $\frac{2}{3} [IP - MDS]$
 = Dynamic Range.

$$-114 + NF + \log\left(\frac{BW}{10^6}\right) = \frac{2}{3} [IP - MDS]$$

Swept Tune FFT-based

① Real time ② less expensive oscilloscope ③ Wide bandwidth

④ noise ↓ ↓ but phase measurement?
transient

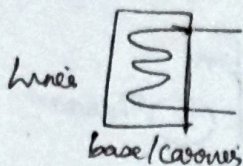
Type of Strain Gauges

- 1) unbonded metal S-G
wire transducer appⁿ.



+ Wheatstone bridge

- 2) Bonded wire : stress analysis + transducers



Base = bakelite, deflon

- 1) high G_f
- 2) high R (lead, hermetic) ~ 1000
- 3) low α'
- 4) low hysteresis
- 5) linear
- 6) good freq. response

Nichrome, Constantan.

80 20 Ni 45
Cu 55

Iso-elastic : Ni Cr Mo
36 8 0.5

- 3) Bonded metal foil

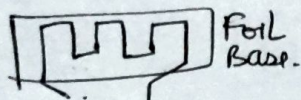


photo etching

stress + transducer

higher heat dissipⁿ
(surface area)

+ better bonding

fat end turns : $\downarrow$ transverse sensitivity
(spurious) $\uparrow$ longitudinal ".

- 4) vacuum deposited / Thin metal SG
- 5) sputter "

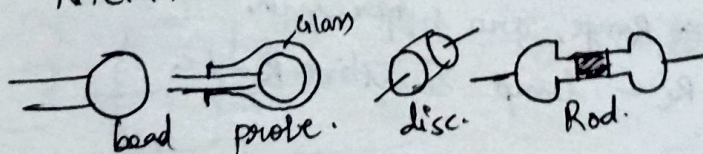
6) semiconductor
+ PZT active -

- i) sensitive G_f
- ii) temp (f)
- 2) $\downarrow$ hysteresis
- ii) linearity $\frac{\Delta}{R} = A\sigma + B\sigma^2$
- 3) fatigue life
- 4) small
- ii) $\frac{\Delta}{R}$

Thermistors

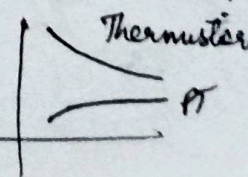
NTCT PTC

sintered mixture of metallic oxides $Mn, Co, Ni, \dots$



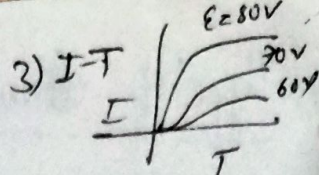
$$R_{T2} = R_{T1} \exp \left(\beta \left(\frac{1}{T_2} - \frac{1}{T_1} \right) \right)$$

Can be linearised in small range.

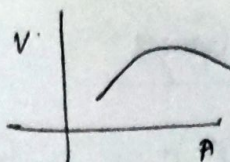


1) R-T

2) V-I



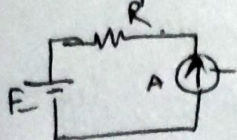
Self heat $\Rightarrow$ sensitive to anything that changes the rate at which



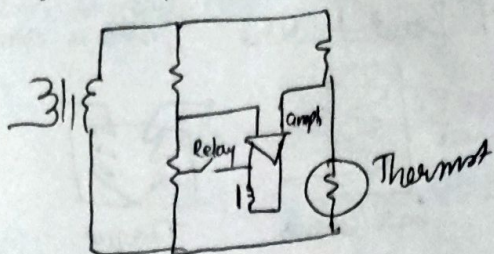
heat is conducted away

Apps

1) Temp Measurement

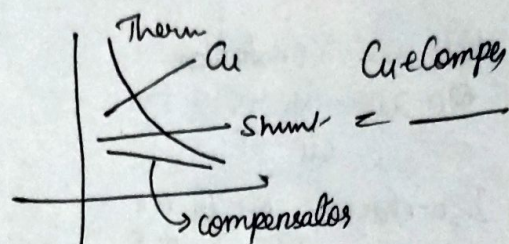
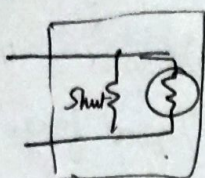
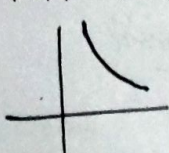


2) Temp control



3) Temp compensation

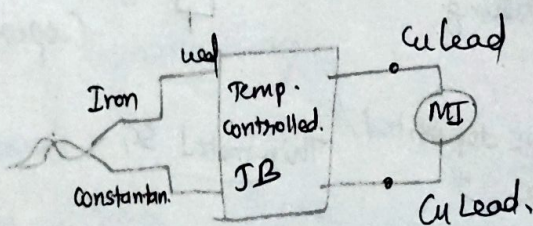
NTP-PTC.



Compact, rugged, cheap, stable, $\frac{1}{2}$

Thermocouple

$$Emf = a40 + b(40)^2$$



Cu-Constantan.
 $\rightarrow$ residues REDOX

Iron-Constantan
 $\rightarrow$ but corrodes

Chromel-Alumel

Chromel-Constantan

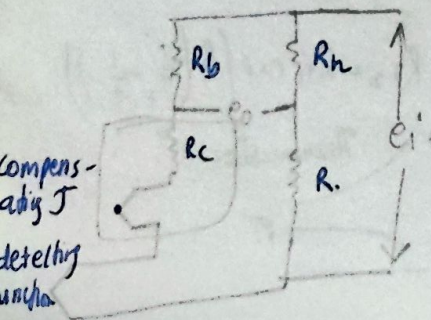
PTT Rh. Rh Irden

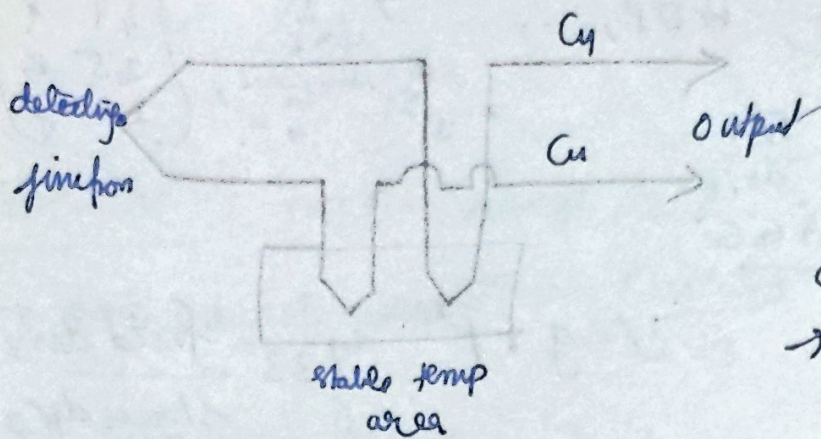
Compensation

1) When temp to be measured are near ambient / inconvenient to reference junction

R = Range, zero suppression

R_c = Temp sensitive Resistor





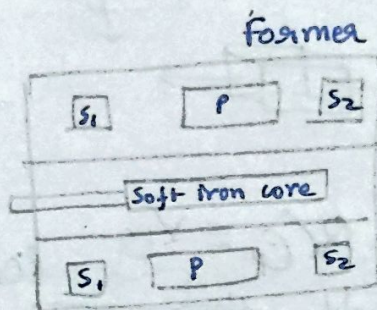
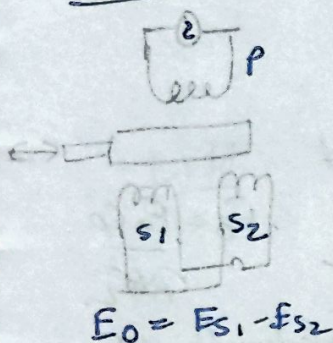
b) Refer function compensation

c) Lead compensation
→ use same wire material

adv: $\pm$, $\downarrow$ --- : accuracy, protection, wiring

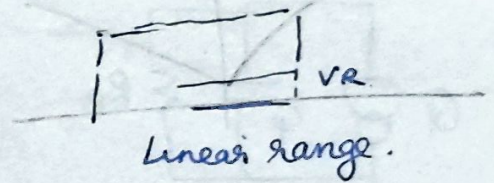
LM335, LM34, AD592

LVDT



at core null $E_{S1} = E_{S2}$

Core - high permeability
Primary = AC current
 E_0 $\phi = 60^\circ$

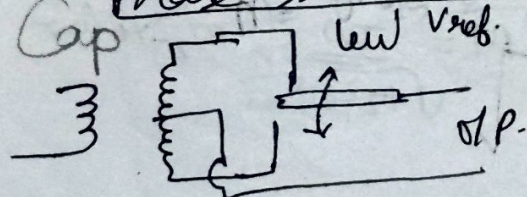


Residual = harmonics in V_i , iron core harmonics

adv: Range, rugged, sensitivity, friction/isolation, electrical
external immune, low hysteresis

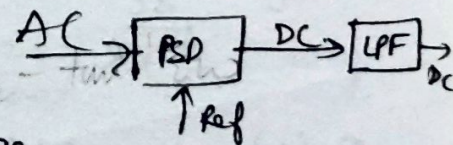
Disad B stray, vibration, AC or PSD iff DC, temp, freq resp

Phase sensitive demodulator

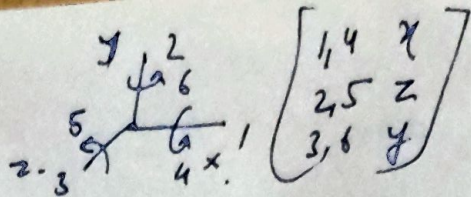


Chopper

or 4-diode bridge



Piezo Rochelle salt, ADP, ..



$$q = df, \quad E = \frac{F/A}{\Delta t/t}$$

$$= dEA' \frac{\Delta t}{t} = V \cdot \frac{A' \cdot \Delta t}{t}$$

$$V = \left(\frac{d}{\Delta t} \right) \Delta t$$

$$V = g + P. \quad g_{33} = \frac{\text{field } d\text{m}^3}{\text{stress } \text{dyn/cm}^2}$$

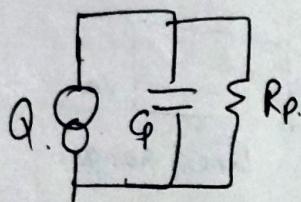
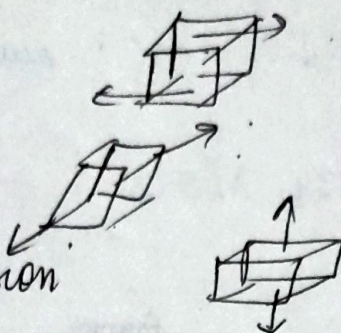
mode

1) Thickness shear

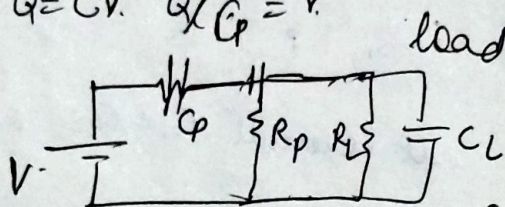
2) Face shear

3) Thickness expansion

4) ~~Face~~ expansion
transverse.

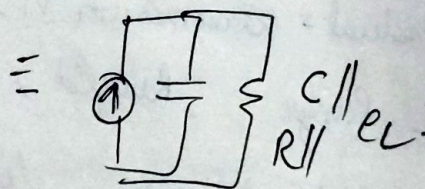
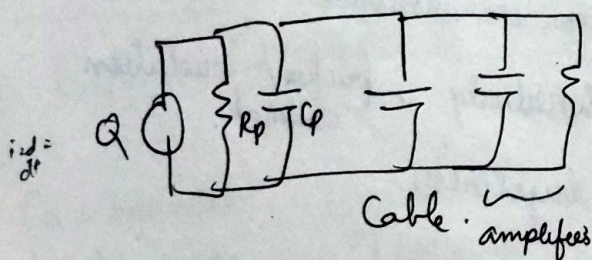


$$Q = CV. \quad Q/C_p = V$$



$$F_0 = \frac{E_0 C_p}{C_p C_L}$$

(medium/high freq)



$$i = dq/dt$$

$$= k \frac{dx_i}{dt}$$

$$e_i = \frac{1}{\frac{1}{R} + R} \cdot i = \frac{R}{R(1+1)} \cdot i$$

$$e_i(s) = \frac{RK}{R(1+1)}, \quad K = KC \Rightarrow \frac{K}{R(1+1)} \quad M = \frac{TW}{\sqrt{1+2\omega^2}}, \quad \pi/2 - \tan^{-1} \omega$$

$$Z_L = \frac{R_L}{1 + j\omega C_L R_L}$$

ADC-DAC $MSB = range/2$, $LSB = range/2^n$.

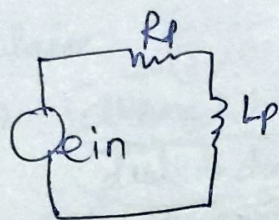
1) RMS value of quantizingⁿ = $\frac{Resolution}{2\sqrt{3}}$

2) Sampling freq = $5f_h$.

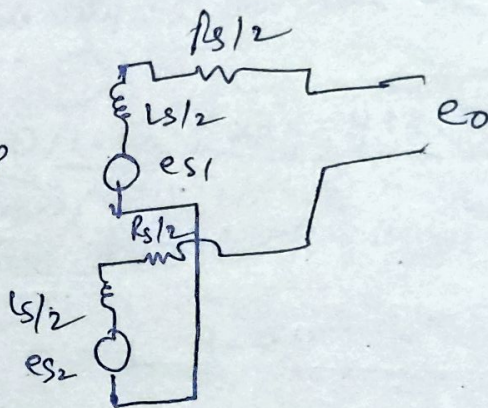
3) aperture time $t_a = \frac{\Delta E}{2\pi f \cdot E_m} = \frac{1}{2^n \cdot 2\pi f}$

4) Dynamic Range = 6ndB, $n = \text{bits}$

$V_{in} R$
 V_{out}
LVDT



$$i_1 = \frac{e_{in}}{sL_p + R_p}$$

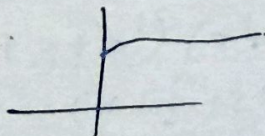
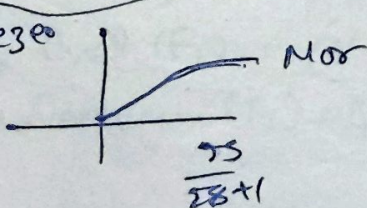


$$e_{s1} = \frac{M_1 di_1}{dt} = \frac{s M_1 e_{in}}{sL_p + R_p}$$

$$e_{s2} = \frac{s M_2 e_{in}}{sL_p + R_p}$$

$$e_o = e_{s1} - e_{s2} = \frac{s e_{in} (M_1 - M_2)}{sL_p + R_p}$$

Piezo



$d/C_f \left(\frac{s \tau B}{s \tau B + 1} \right) \left(\frac{\tau B}{2B + 1} \right)$ make τ large